CSE 250B Quiz 1

Tuesday January 14, 2014

Instructions. Do this quiz in partnership with exactly one other student. Write both your names at the top of this page. Discuss the answer to the question with each other, and then write your joint answer below the question. Use the back of the page if necessary. It is fine if you overhear what other students say, because you still need to decide if they are right or wrong. You have seven minutes. The maximum score is three points.

Question. Consider training a logistic regression classifier when each example has just one feature value. In other words, each instance is a scalar outcome $x \in \mathbb{R}$. As usual, each label is a binary outcome $y \in \{0, 1\}$. Consider the following training set of four labeled examples:

$$\begin{array}{c|c} X & Y \\ \hline 0 & 0 \\ 10 & 0 \\ 10 & 1 \\ 0 & 1 \\ \end{array}$$

The logistic regression model is

$$p(Y = 1|X = x) = \frac{1}{1 + \exp{-[\alpha + \beta x]}}$$

For the training set above, what are the trained values of α and β ? Justify your answer with an informal explanation, not with a detailed calculation.

Answer. Each training value x appears once with each possible label y, so there is no visible dependence of the random variable Y on X. Hence, the coefficient β of x must be zero. Each label value is equally likely, so for all x we should have p(Y = 1|X = x) = 0.5. To achieve this, the exponential must equal one, which implies that α must be zero.

Additional note. The training data do not show that the random variables X and Y are independent, or even that they have no linear correlation (which is a much weaker condition). No finite dataset can prove independence or lack of correlation. However, this dataset provides no evidence in favor of correlation.