CSE 250A Quiz 7

Tuesday November 20, 2012

Instructions. You should do this quiz in partnership with exactly one other student. Write both your names at the top of this page. Discuss the answer to the question with each other, and then write your joint answer below the question. It is ok if you overhear other students' discussions, because you still need to decide if they are right or wrong. You have seven minutes.

Consider the following infinite sequence of observations over an alphabet of 26 alternative values: *aabbcc . . . yyzzaabbcc . . . yyzzaabbcc . . .*
(a) Explain how this sequence of observations does *not* satisfy the first-order Markov property.
(b) Nevertheless, describe the CPTs of a hidden Markov model (HMM) that outputs the sequence of observations above with probability one. Your description should be clear, but it can be informal. In particular, you may use the notation "..." (ellipsis).

Answer. The probability of the next letter depends on the two previous letters, but the first-order Markov property requires this probability to depend only on the single previous letter.

Let the HMM have 52 states, called $\{a, A, b, B, \ldots, z, Z\}$. Define

$$\pi_\alpha = p(S_1 = \alpha) = I(\alpha = \text{``}a\text{''}).$$

Define

$$p(O_t = \phi | S_t = \alpha) = 1 \text{ iff } (\alpha = \text{``}a\text{''} \text{ or } \alpha = \text{``}A\text{''}) \text{ and } \phi = \text{``}a\text{''} \text{ and so on.}$$

Define

$$p(S_{t+1} = \beta | S_t = \alpha) = 1 \text{ iff } (\alpha = \text{``}a\text{''} \text{ and } \beta = \text{``}A\text{''}) \text{ or } (\alpha = \text{``}A\text{''} \text{ and } \beta = \text{``}b\text{''}) \text{ and so on.}$$

Additional comments. By definition, the state transitions of a hidden Markov model satisfy the first-order Markov property. However, increasing the number of hidden states allows each state effectively to encode a history of more than one observation, so the sequence of observations can be more complicated.