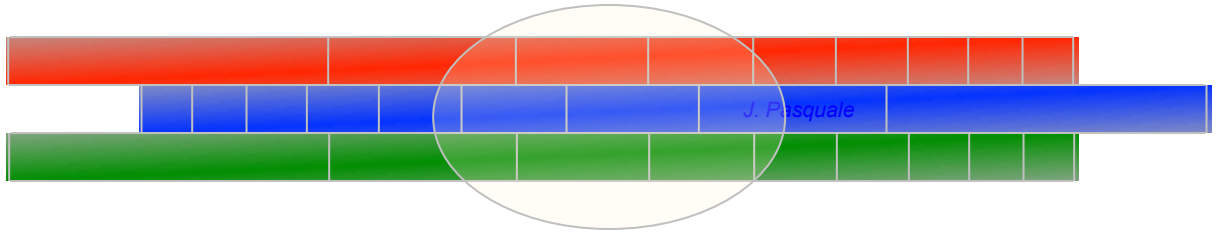
Why Does a Slide Rule Work?



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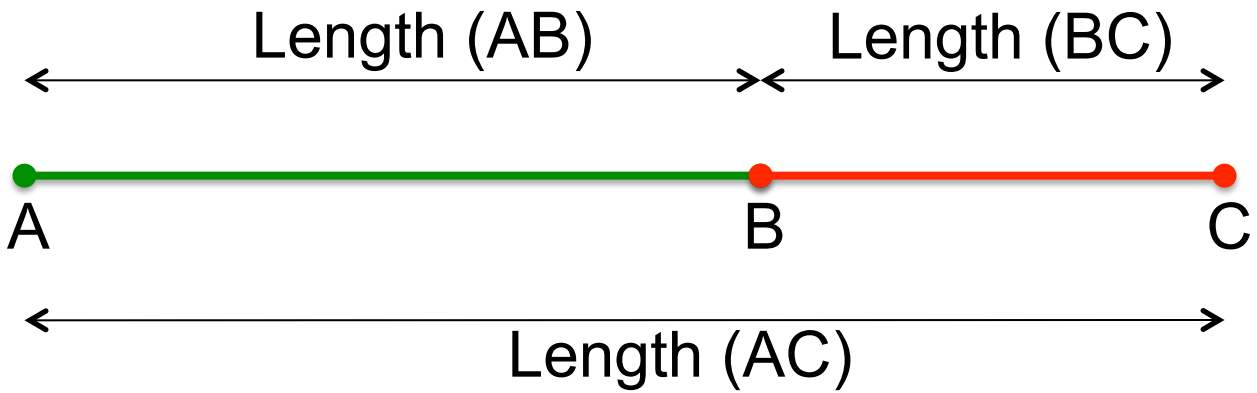
Geometric Basis

- Two in-line line segments: AB and BC

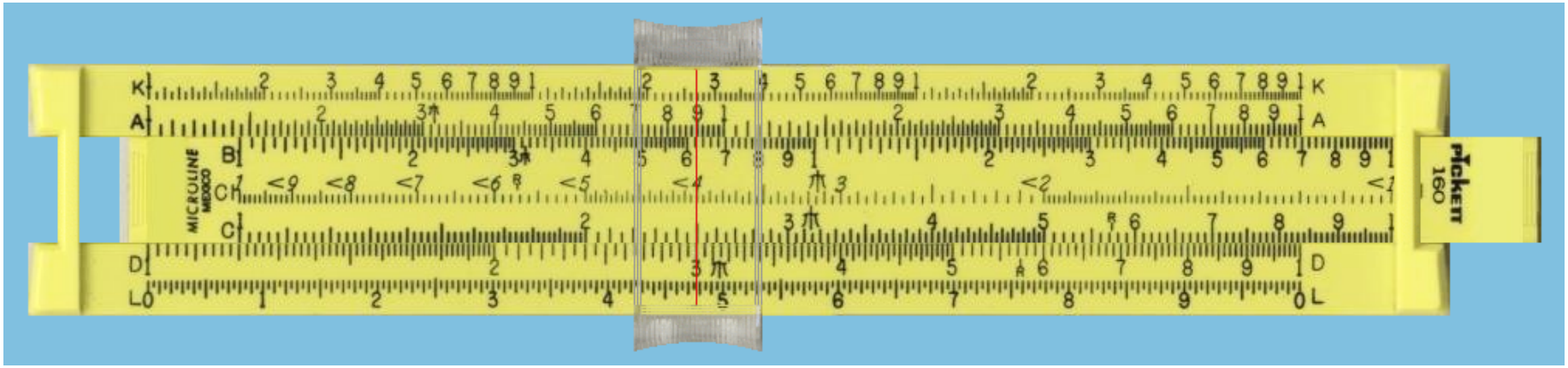
Geometric Basis



- Two in-line line segments: AB and BC
- Sum of lengths = length of sum

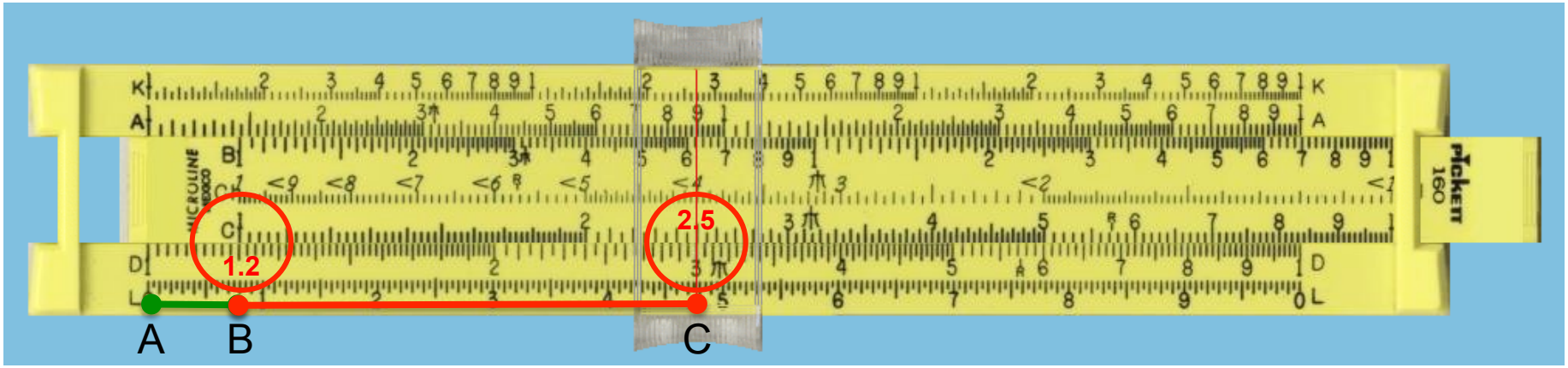
  L(AB) + L(BC) = L(AB+BC) = L(AC)

Slide Rule Scales



- Multiple scales
- Represent various functions
- Parallel to each other

Slide Rule Scales

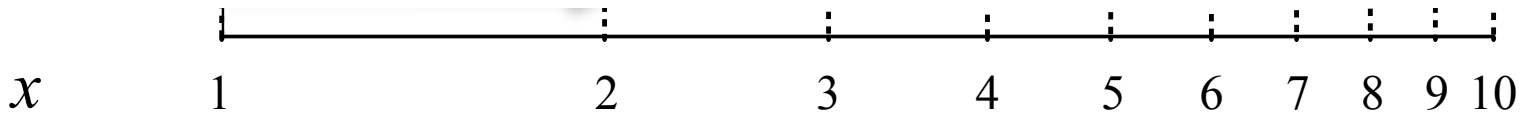


- Slide allows us to add lengths
- But labels are not lengths!

What is a Slide Rule Scale?

- Scale: finite line with graduated marks

What is a Slide Rule Scale?



- Scale: finite line with graduated marks
- Each mark corresponds to a value *x*

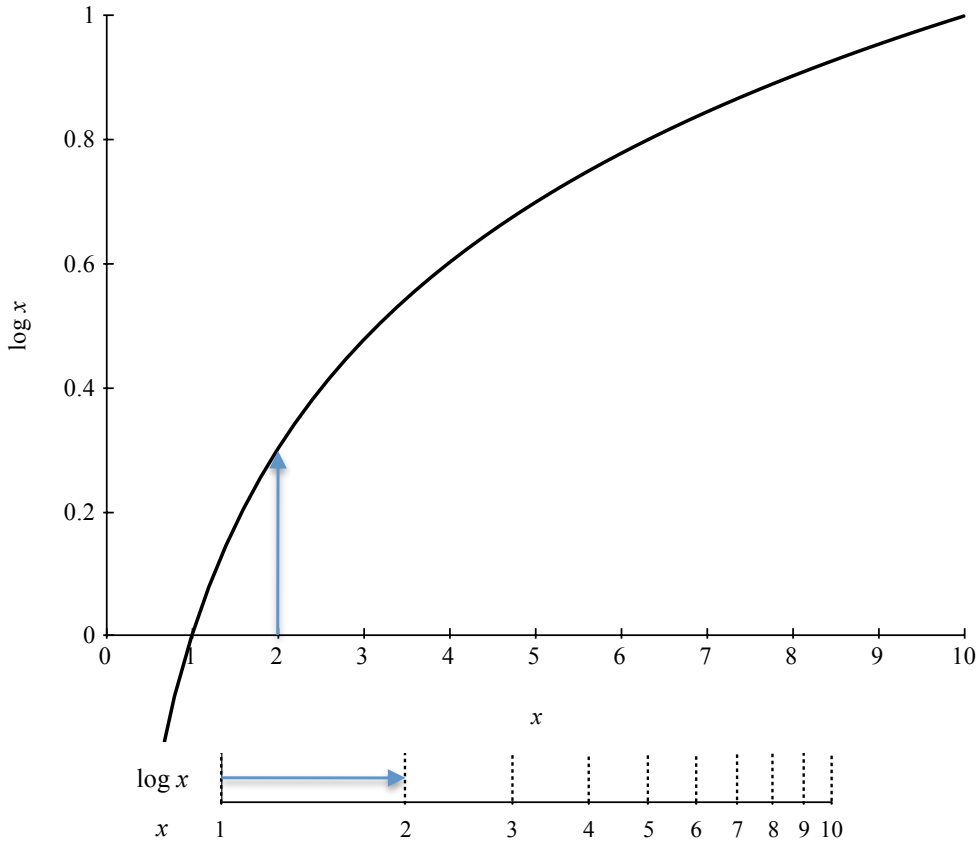
What is a Slide Rule Scale?

- Scale: finite line with graduated marks
- Each mark corresponds to a value *x*
- Located at a distance *f(x)* from origin

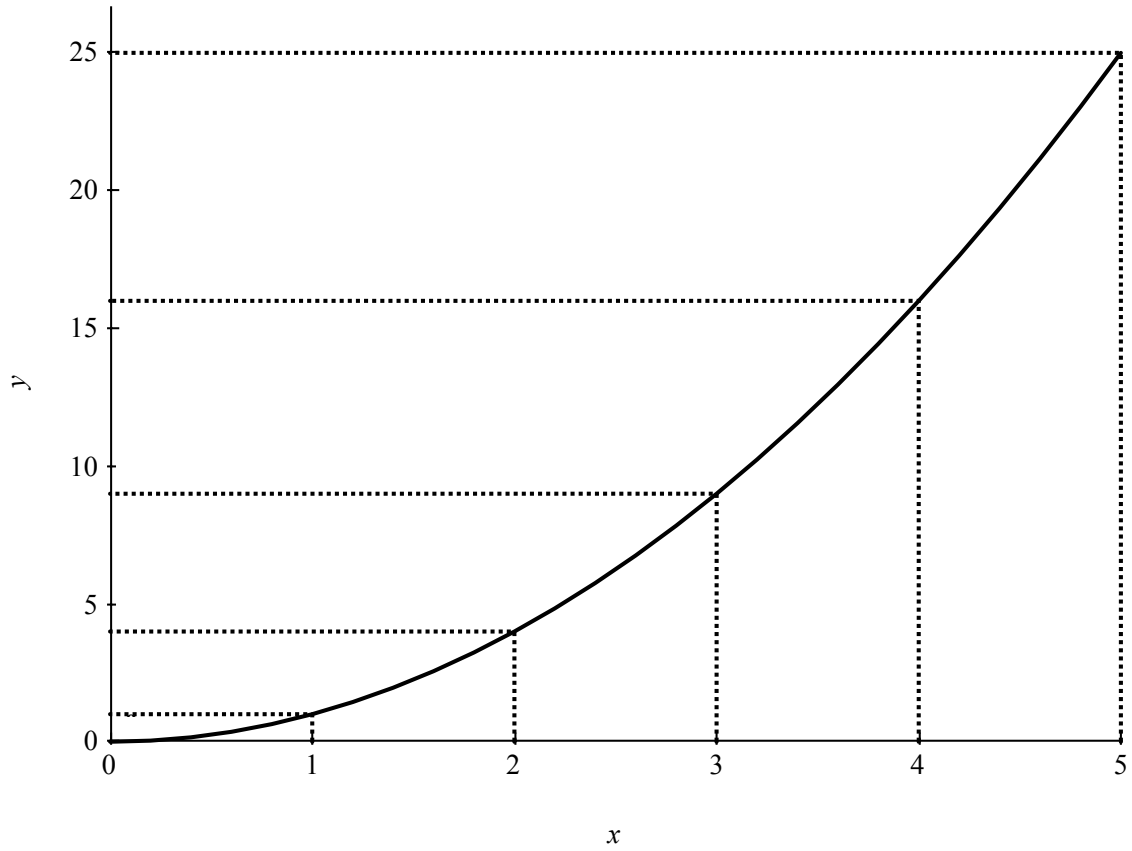
What is a Slide Rule Scale?

- Scale: finite line with graduated marks
- Each mark corresponds to a value *x*
- Located at a distance *f(x)* from origin
- *f(x)* is the *function of the scale*

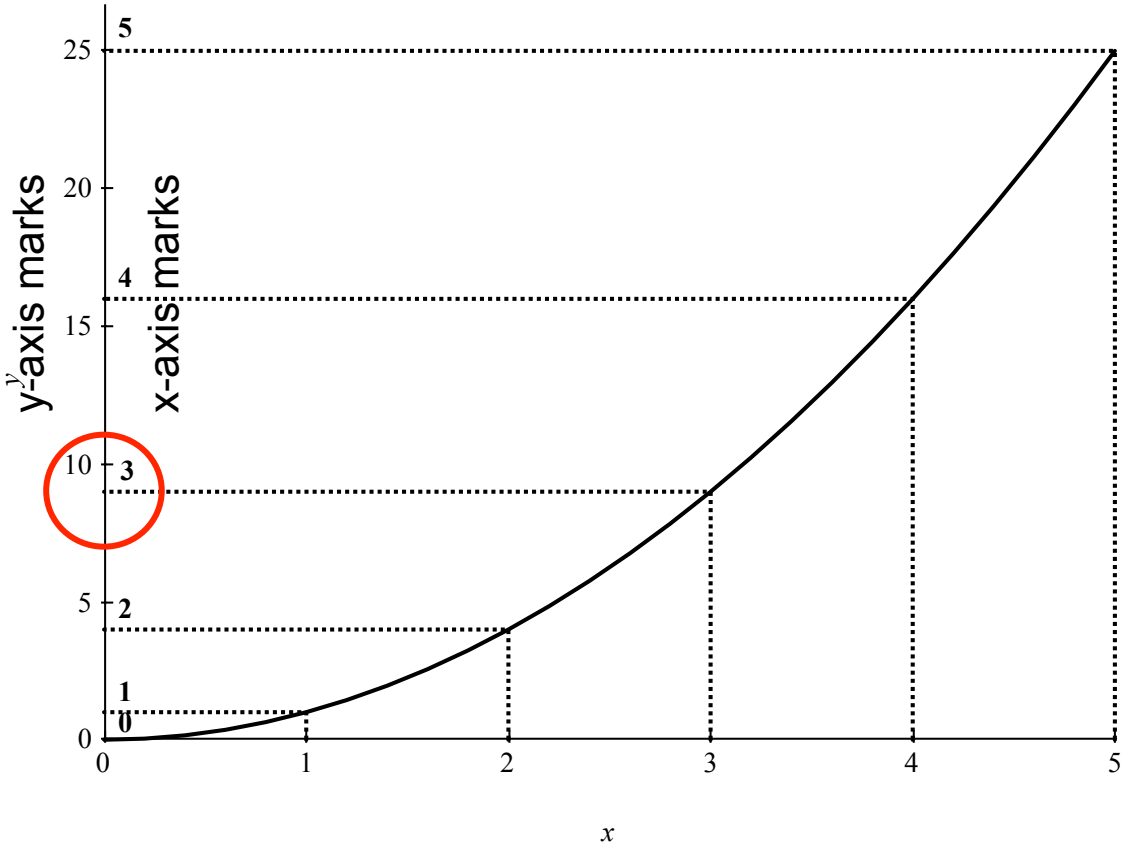
From 2 Dimensions to 1



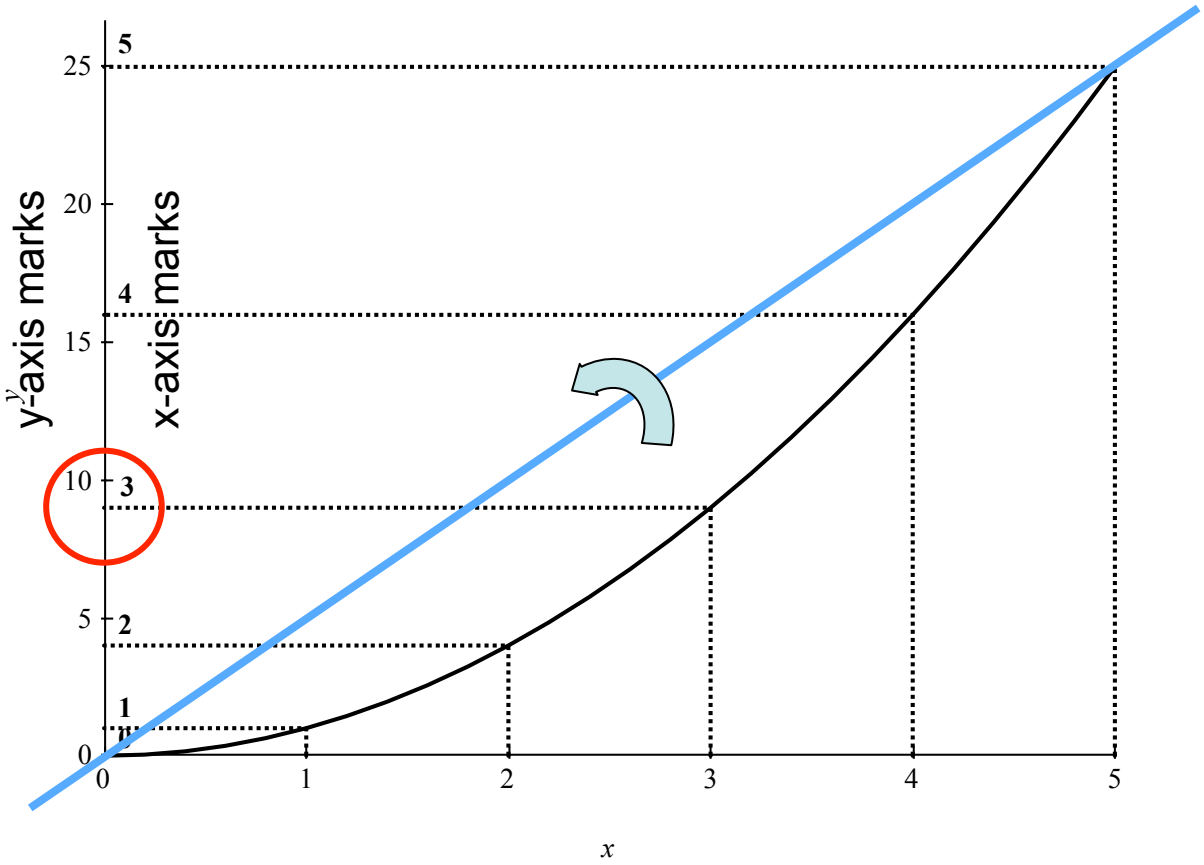
Example: Graph for $f(x) = x^2$



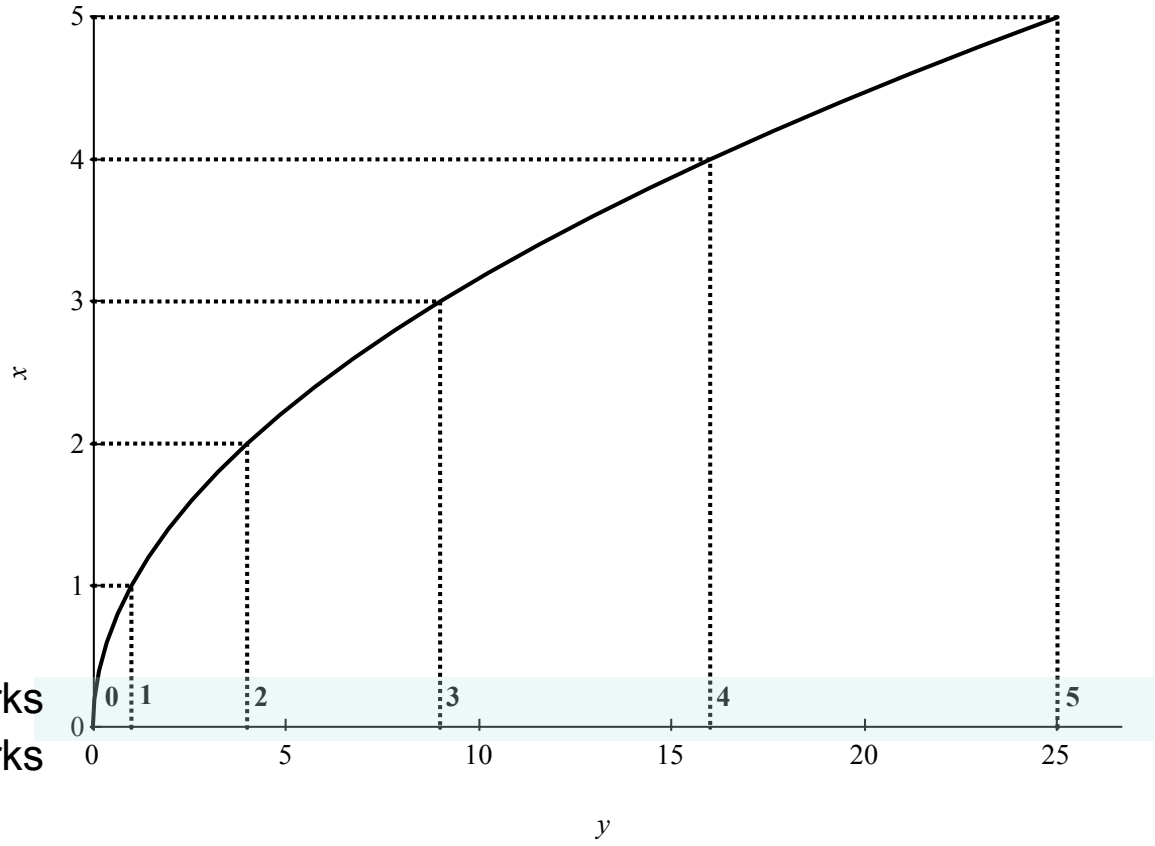
Copy x-axis Marks to y-axis



Next, Flip About Its Diagonal



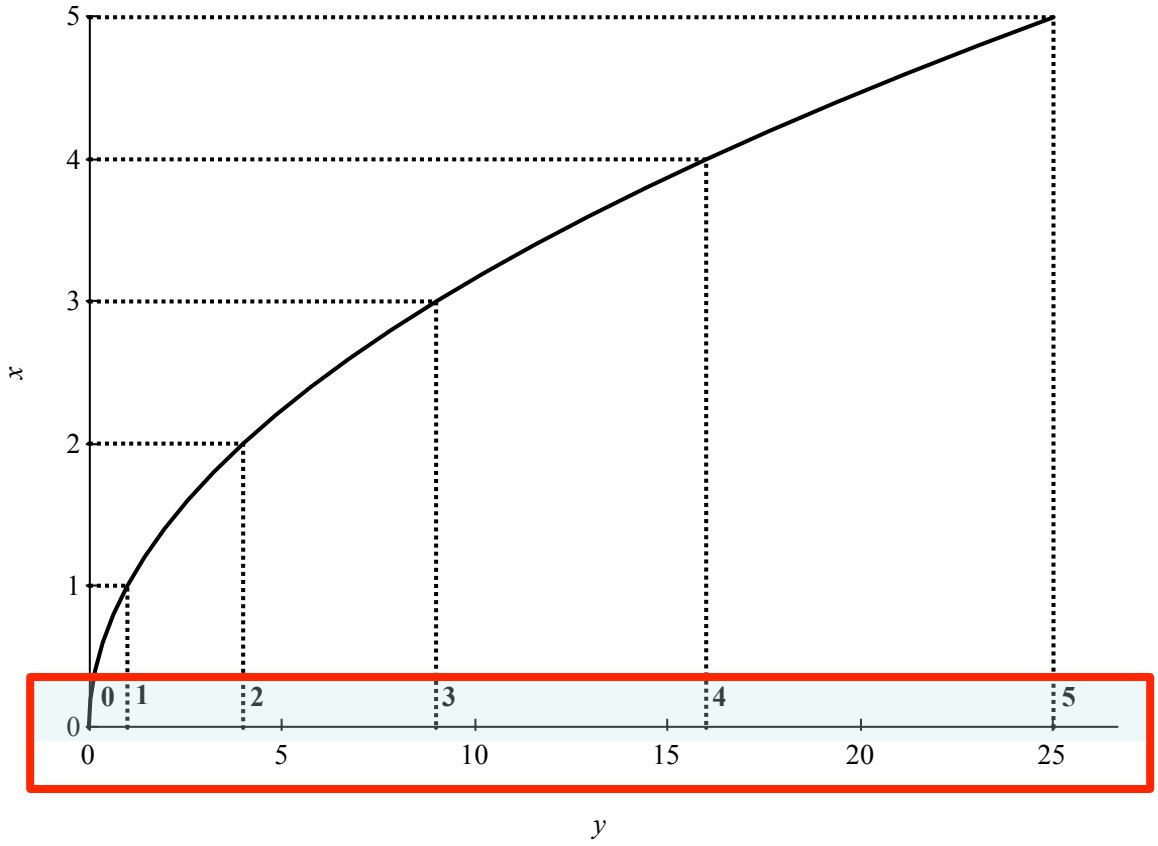
Resulting Graph



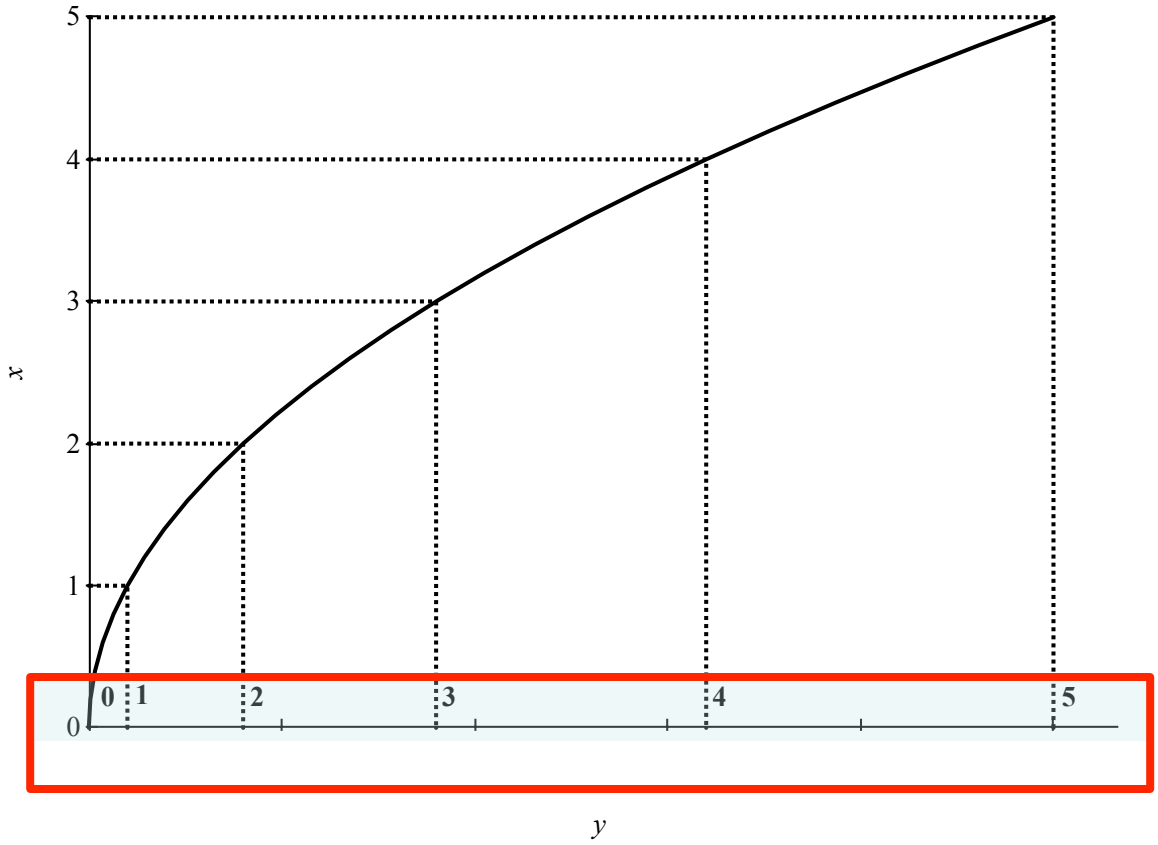
x-axis marks

y-axis marks

Focus on New x-axis



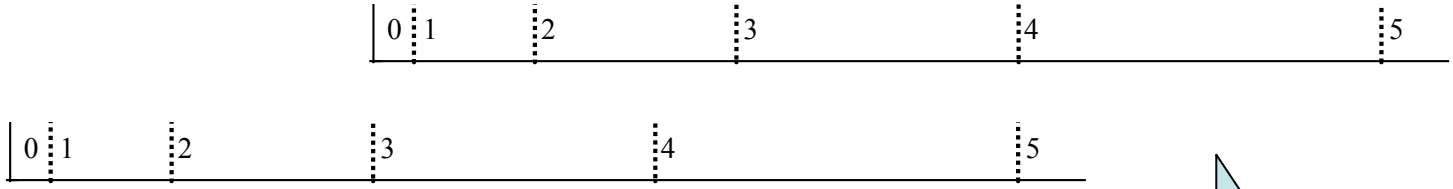
Remove Original y-axis Marks

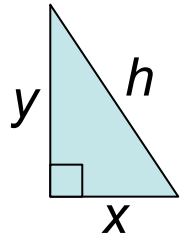


Resulting Scale

- Scale for $f(x) = x^2$
- Notice
  - Each mark is labeled by x
  - Distance from left end is $f(x) = x^2$

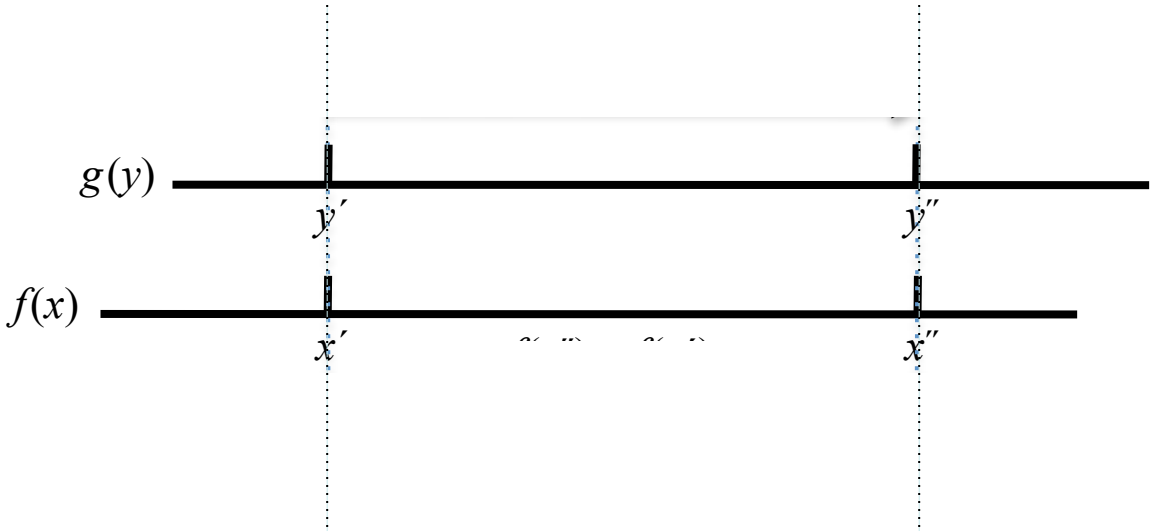
Calculating Using Scales



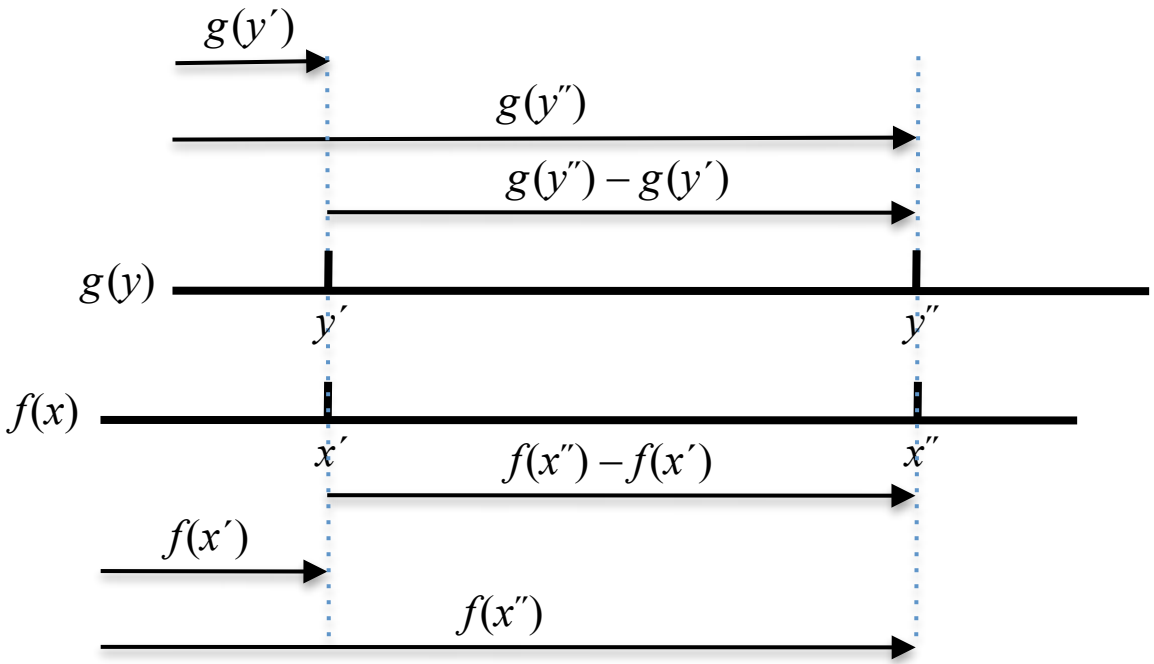


- 2 scales, each $f(x) = x^2$
- Resulting function: $f(x,y) = \sqrt{x^2 + y^2}$
- In the above, $\sqrt{3^2 + 4^2} = 5$

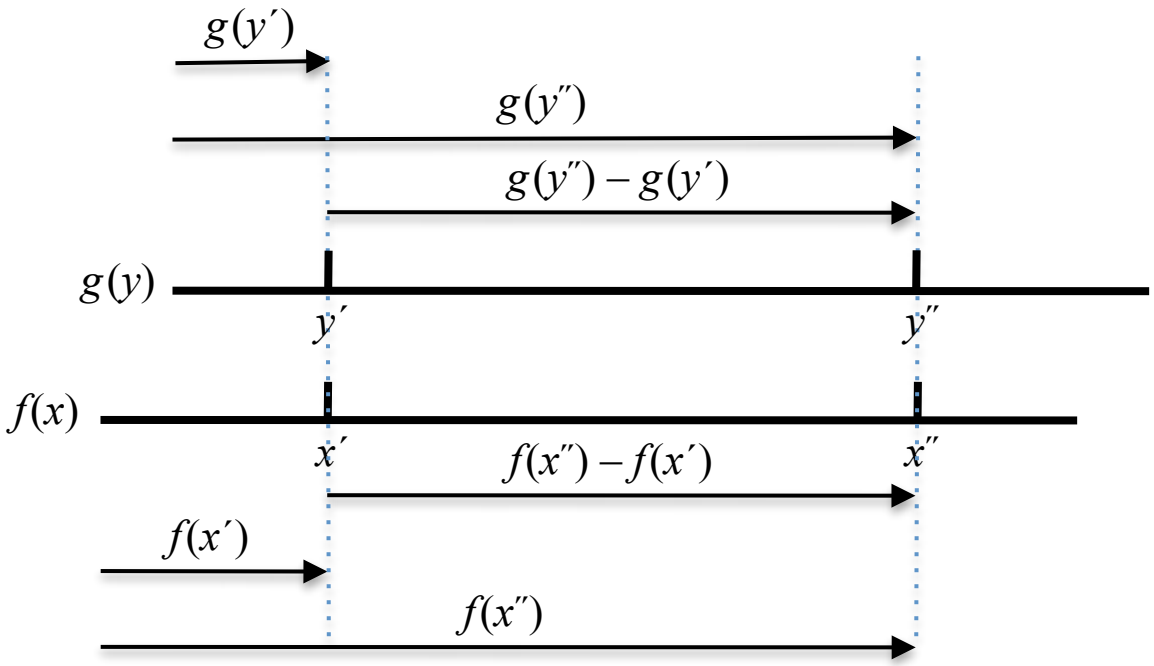
Key Property of Sliding Scales



Key Property of Sliding Scales



Key Property of Sliding Scales



$$f(x'') - f(x') = g(y'') - g(y')$$

Mathematical Principle of the Slide Rule

- A slide rule with two scales that are defined by the functions *f*(*x*) and *g*(*y*) can calculate any function of the form:

$$h(x,y,z) = f^{-1}(f(x) + g(y) - g(z))$$

Some Examples: Addition

$$h(x,y,z) = f^{-1}(f(x) + g(y) - g(z))$$

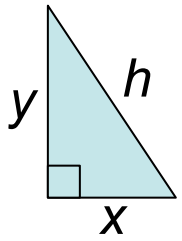
- Given $f(x) = x$, $g(y) = y$

$$h(x,y,0) = f^{-1}(x + y - 0) = x + y$$

Hypotenuse of Right Triangle

$$h(x,y,z) = f^{-1}(f(x) + g(y) - g(z))$$

- Given $f(x) = x^2$, $g(y) = y^2$

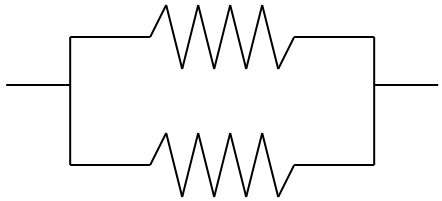


$$h(x,y,0) = f^{-1}(x^2 + y^2 - 0) = \sqrt{x^2 + y^2}$$

Resistors in Parallel

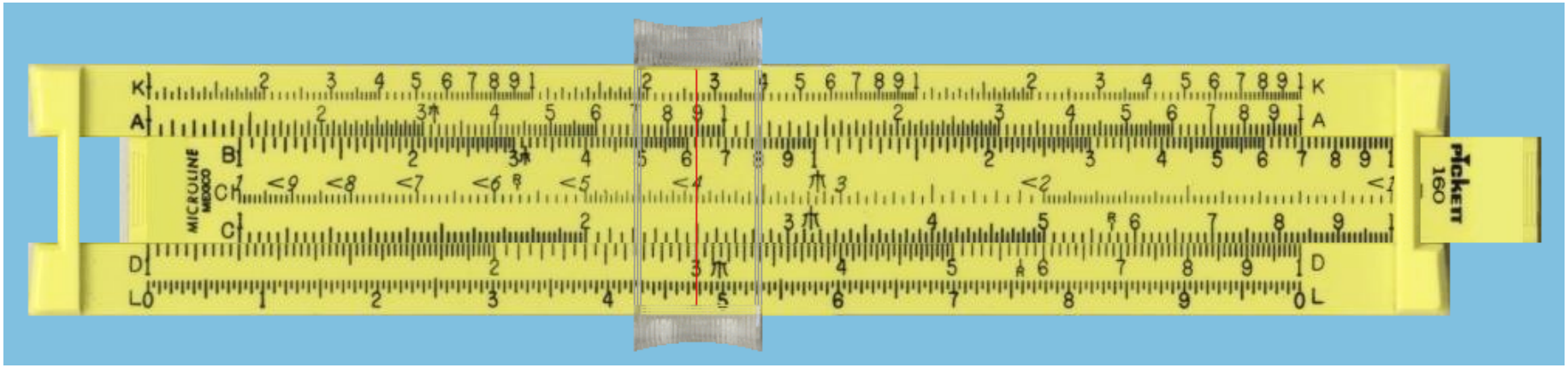
$$h(x,y,z) = f^{-1}(f(x) + g(y) - g(z))$$

- Given $f(x) = 1/x$, $g(y) = 1/y$



$$h(R_1,R_2,\infty) = f^{-1}\left(\frac{1}{R_1} + \frac{1}{R_2} - 0\right) = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

What about Logarithms?



- Most slide rules have logarithmic scales
- Why?
  - Because they lead to a wide range of very useful types of calculations

Multiplication

$$h(x,y,z) = f^{-1}(f(x) + g(y) - g(z))$$

- Given $f(x) = \log x$, $g(y) = \log y$

$$h(x,y,1) =$$

$$f^{-1}(\log x + \log y - 0) = 10^{\log x + \log y} = xy$$

Raising to a Power

$$h(x,y,z) = f^{-1}(f(x) + g(y) - g(z))$$

- Given $f(x) = \log\log x$, $g(y) = \log y$

$$h(x,y,1) =$$

$$f^{-1}(\log\log x + \log y - 0) = 10^{y\log x} = x^y$$

Raising to a Power

$$h(x,y,z) = f^{-1}(f(x) + g(y) - g(z))$$

- Given $f(x) = \log\log x$, $g(y) = \log y$

$$h(x,y,z) =$$

$$f^{-1}(\log\log x + \log y - \log z) = 10^{\frac{y}{z}\log x} = x^{\frac{y}{z}}$$

UCSD Slide Rule Seminar

- Started seminar in 2003
- Students come from all areas
  - Engineering, physical sciences, some humanities

Why Teach about Slide Rules?

- Engineering heritage
- Power of slide rules vs. computers
- Interesting mathematics

Why Are Students Interested?

Why Are Students Interested?

"I have no idea what a slide rule is, but I know engineers used them to build great things"

Why Are Students Interested?

"I thought it would be cool to learn how to use a slide rule and show my friends"

Why Are Students Interested?

"My dad used a slide rule, and I want to learn about it (and show him)"

Topics

- How the slide rule works
- The math behind the slide rule
- Precision vs. accuracy
- How to construct all the scales
- Advanced Topics

Larger Lessons

- Economy of calculating
  - slide rules
  - calculators
  - computers
- Estimation, approximation
- Social value
  - parents, grandparents

Quotes

"My skills of estimation are getting better. I like being engrossed in the calculations, instead of just punching them into my calculator. I make less mistakes, and find I know what I am talking about."

Quotes

"I was in physics class, and the professor explained how tan and sin are close for really small angles. The class didn't show much reaction, but my first thought was "Hey, I learned that from my slide rule seminar."

Quotes

"This slide rule seminar is the only thing saving me from a quarter full of literature writing, and other humanitarian monotony. After hours of 'theory of literature,' I realized I still had slide rule homework. Hurray!"

Quotes

"The slide rule is truly an extension of a person, not something completely separate such as the calculator. I actually had to think before, during, and after getting the answer on the slide rule."

Quotes

"The more I use the slide rule, the greater the insight I have into how ingeniously the scales were put together. I hope I can re-teach my parents how to use it."

Masters of the Slide Rule ‘03

FOR MORE INFO

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Paper is available at

http://cseweb.ucsd.edu/~pasquale/Papers/IM11.pdf