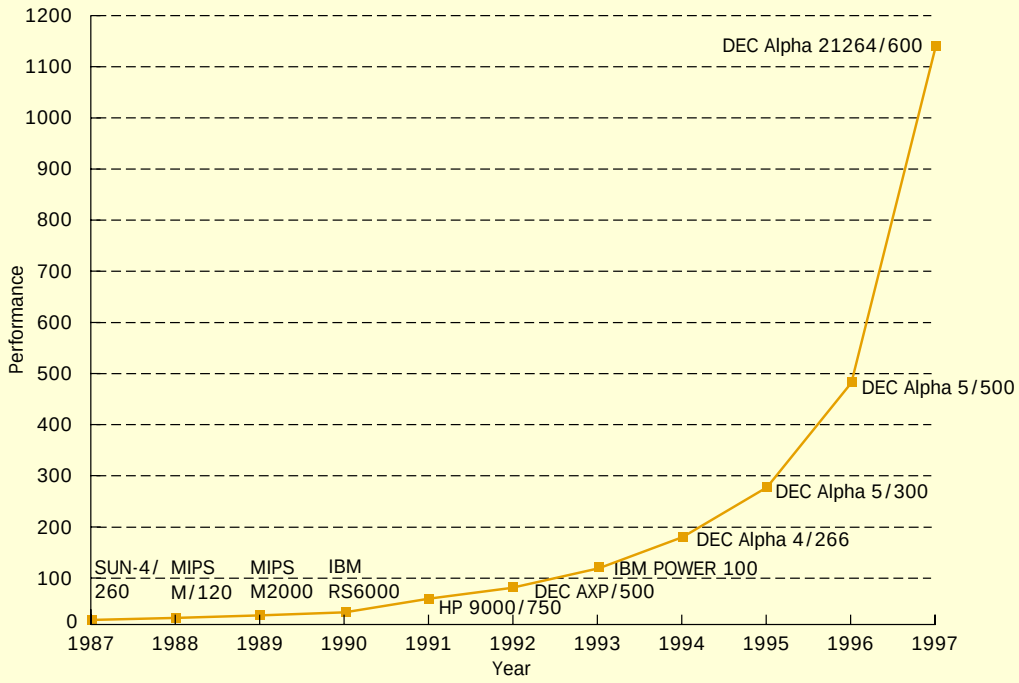
CSE 141

Exponential Growth



Notation for CSE 141

- K = kilo = 1000
- M = mega = 1,000,000
  - Exception: for memory, KByte = 2^{10} Bytes, MByte = 2^{20} Bytes,
  - For BOTEE's, $2^{10} = 10^3$.
- G = giga = 10^9
- T = tera = 10^{12}.
- P = peta = 10^{15}.

- bit – not capitalized
- B or Byte – capitalized
- m = milli = .001
- u (or μ) = micro = 10^{-6}
- n = nano = 10^{-9}

Always keep units (ns, KB, $, ...) around in calculations.
They make a good check.

News from the NYTimes (June '96)

"When a computer runs out of [RAM memory], modern operating systems automatically use the memory on the hard drive. But today's hard drives retrieve data at speeds of about 10 milliseconds (millionths of a second). That seems fast until you consider that modern RAM can do this at 60 nanoseconds (billionths of a second), more than 150 times as fast."

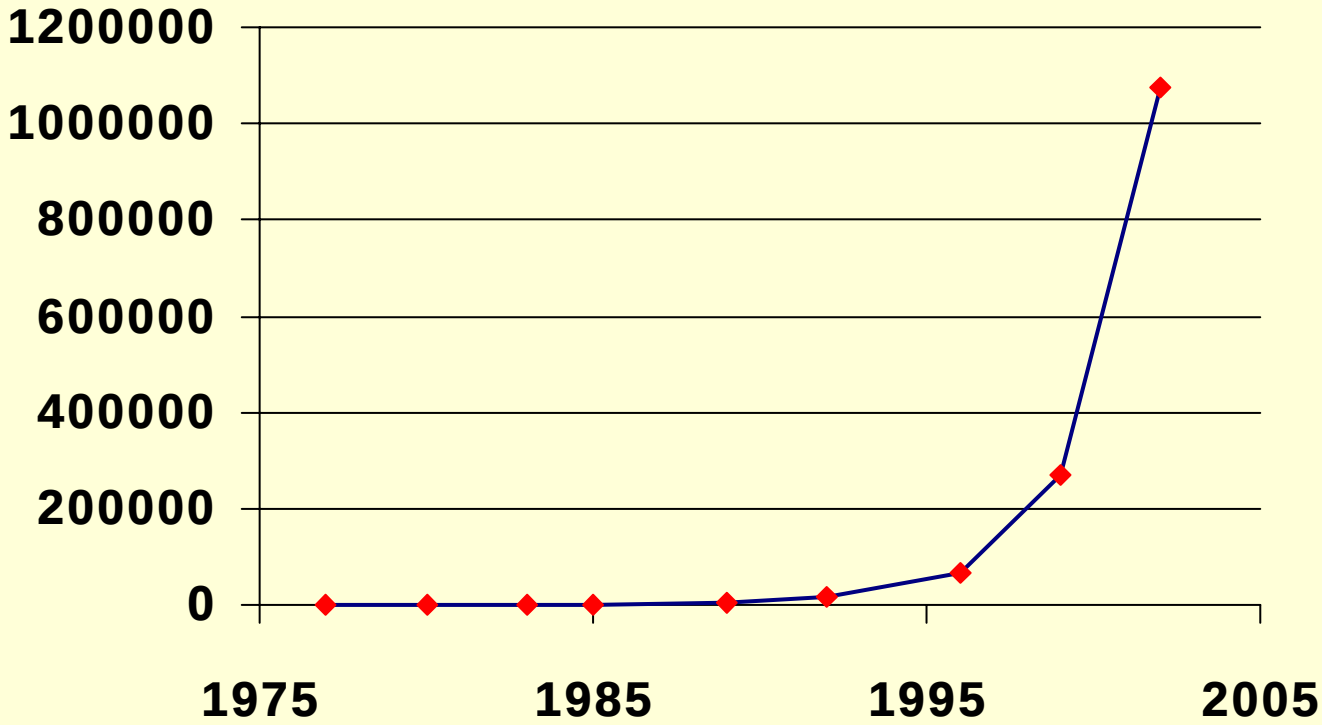
What's wrong with the above statement??

- 1 Extra Credit point to first person to show me obvious innumeracy in current reputable newspaper.
  - Limit: 2 points per person.

DRAM Capacity

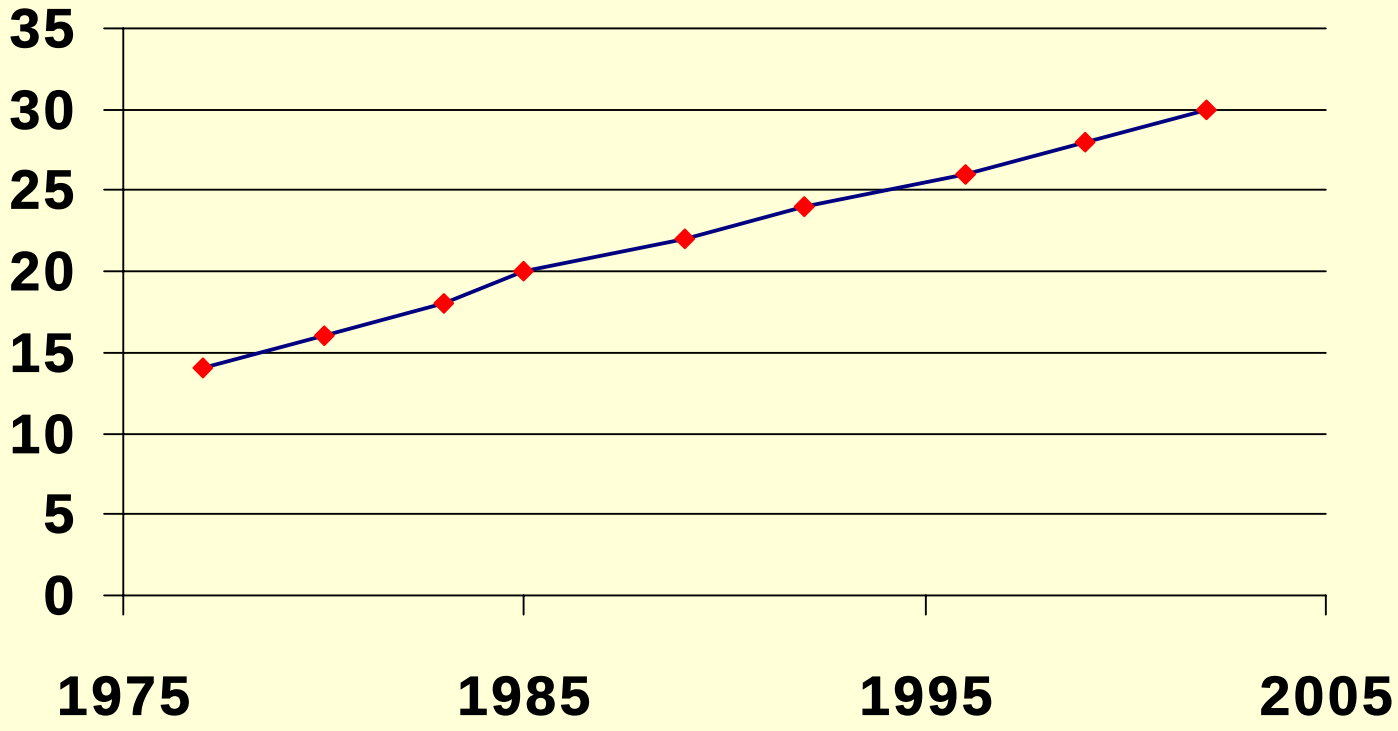
- Graph shows capacity (in thousands of bits) of largest DRAM chip.

Has all growth has been in the last decade??



Logarithmic Scale

y axis is log (base 2) of capacity (in bits) of largest DRAM chip



DRAM growth rule

gets 4 times larger

"DRAM capacity quadruples every three years"

- Remarkably consistent over last 25 years.
  - Introduction of 1 Mbit chip came a year early.
  - 64 Mbit chip (and larger ones?) coming a year late.
- To some extent, a self-fulfilling prophecy.
- Improvement due to smaller lithography and to increased chip size.
  - Chips aren't *profitable* until they are small.

The news from www.news.com

- "16 Mbit DRAM can store information from 128 newspaper pages."
  - 2^{24} bits / 2^{7} pages = 2^{17} bit/page
- "4 Gbit DRAM can store 16,000 newspaper pages of information."
  - 2^{32} bits / 2^{14} pages = 2^{18} bit/page

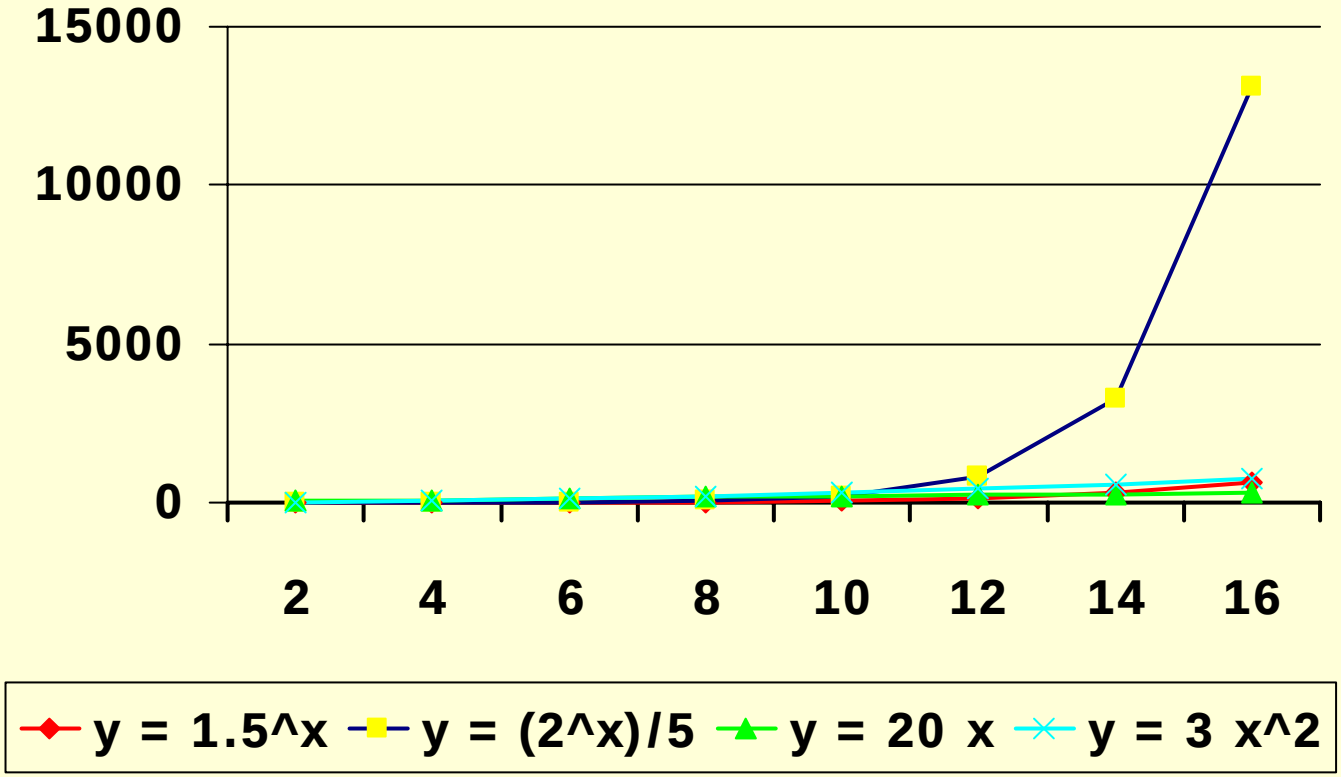
You should be able to do power-of-2 computations like this very easily.

Are newspaper pages getting larger?

Perhaps bits hold less information than in "good old days"??

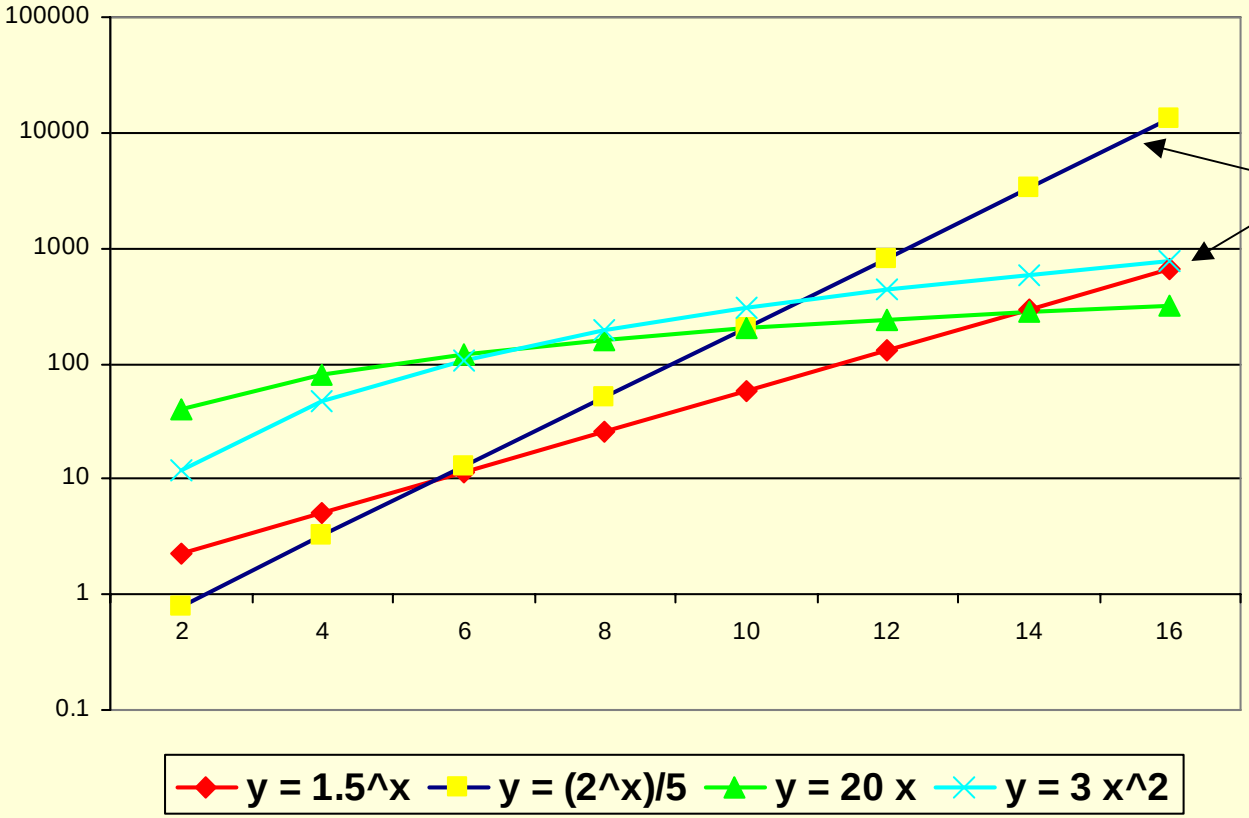
Using a log scale graph

- Here are 4 functions drawn as ordinary graph.



Using a log scale graph

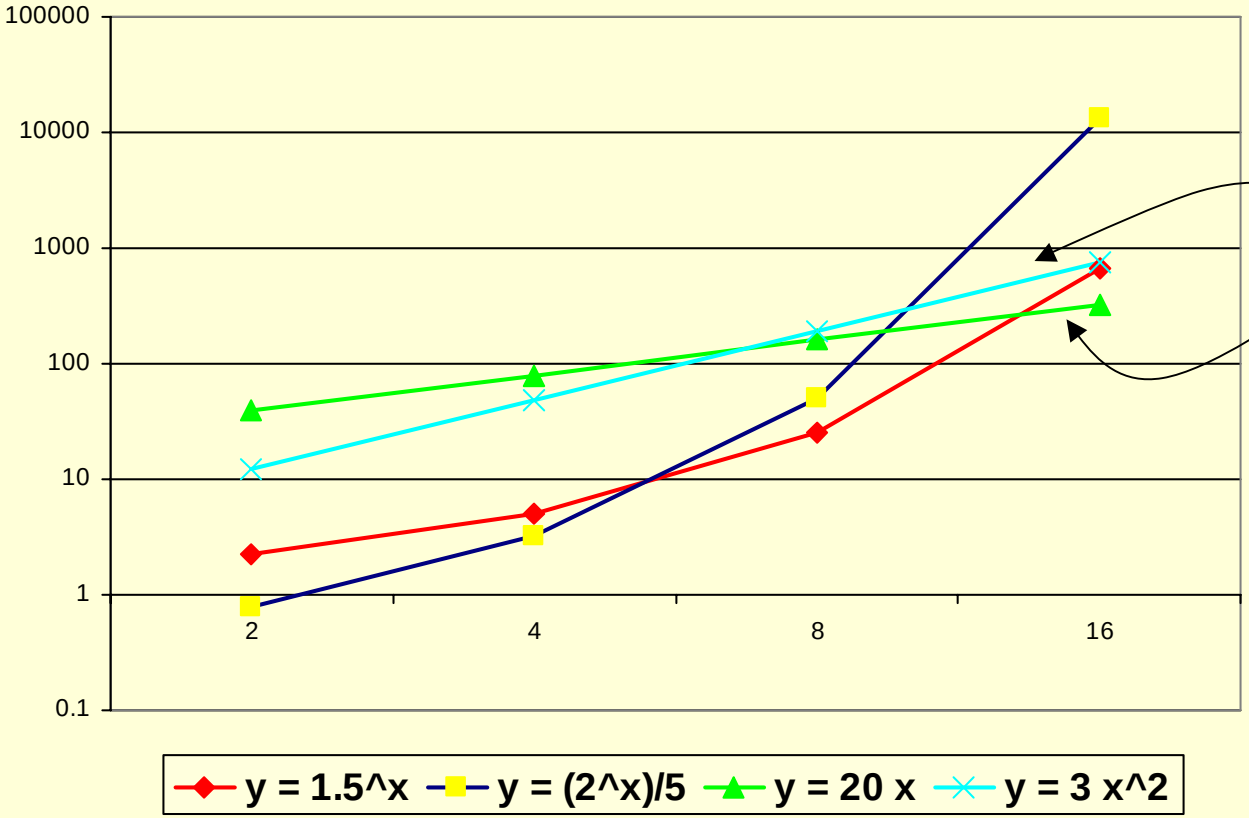
- Good for exponential growth.
  - slope is base of exponent (y-intercept is multiplier)



Red and Blue lines grow exponentially (they are straight)

Aside: what about log-log graph?

- Good for polynomial growth.
  - slope is exponent (if scales are same, which they aren't here)



Cyan and green lines grow polynomially (they are straight)

Larry's "Rule of 72"

"Something that grows at x% a year will double *about* every 72/x years."

Examples:

- If you get 6% interest on bond, value will double in 72/6 = 12 years.
- If world population grows by 2% a year, it will double every 72/2 = 36 years.
- If DRAM access time drops by 9% a year, it will take 72/9 = 8 years to drop by half.
- If DRAM capacity doubles every 18 months, it increases by an average of 72/18=4% per month.

How close is rule of 72?

Accurate within 5% for p up to about 18 .

For p < 4, “Rule of 70” is a little better

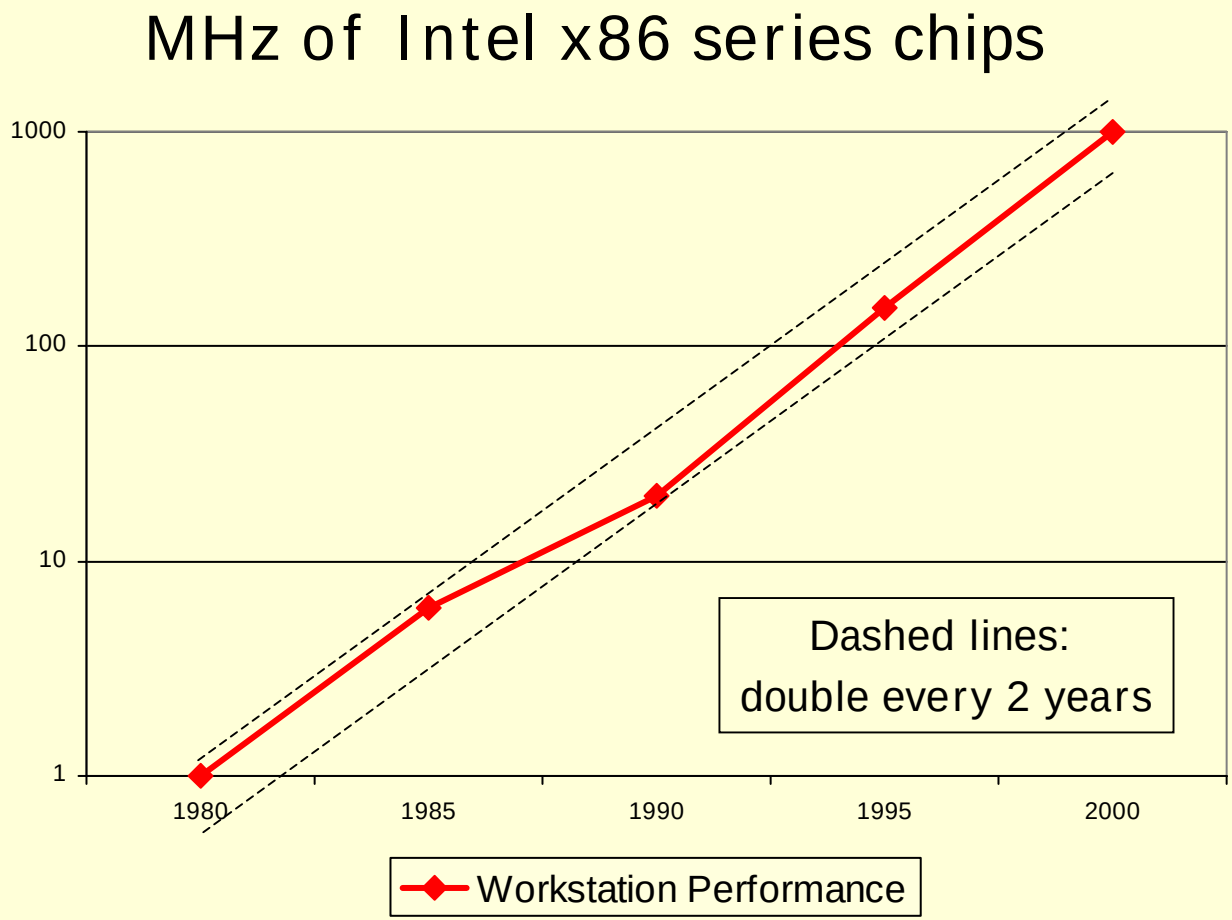
For p > 20, approximation is lousy!

p	1	2	3	6	12	18	24
72/p	72	36	24	12	6	4	3
growth after 72/p steps	2.05	2.04	2.03	2.01	1.97	1.94	1.91

Justification of rule of 72

- Suppose $x(t)$ is p% larger than $x(t-1)$.
  - $x(1) = x(0)\,(1+p/100)$
  - $x(2) = x(1)\,(1+p/100) = x(0)\,(1+p/100)^2$
  - more generally, $x(t) = x(0)\,(1+p/100)^t$
- From calculus: $e = \lim_{x\to\infty} (1+1/x)^x$.
  - So for "large enough" x, $e \approx (1+1/x)^x$.
  - Choose x so that $1/x = p/100$ (i.e. $x = 100/p$)
  - Then $(1+p/100)^t = (1+1/x)^t = (1+1/x)^{xt/x} \approx e^{t/x} = e^{tp/100}$.
  - So if $x(t)/x(0) = 2$, then $(1+p/100)^t = 2$, so $e^{tp/100} \approx 2$.
  - Thus, $tp/100 \approx \ln 2 = .69314$, i.e. $tp \approx 69.314...$
  - I use 72 since it's easy, and closer for medium-sized p.

Clock speed on a logarithmic scale ...



Aside: a 30x change is BIG

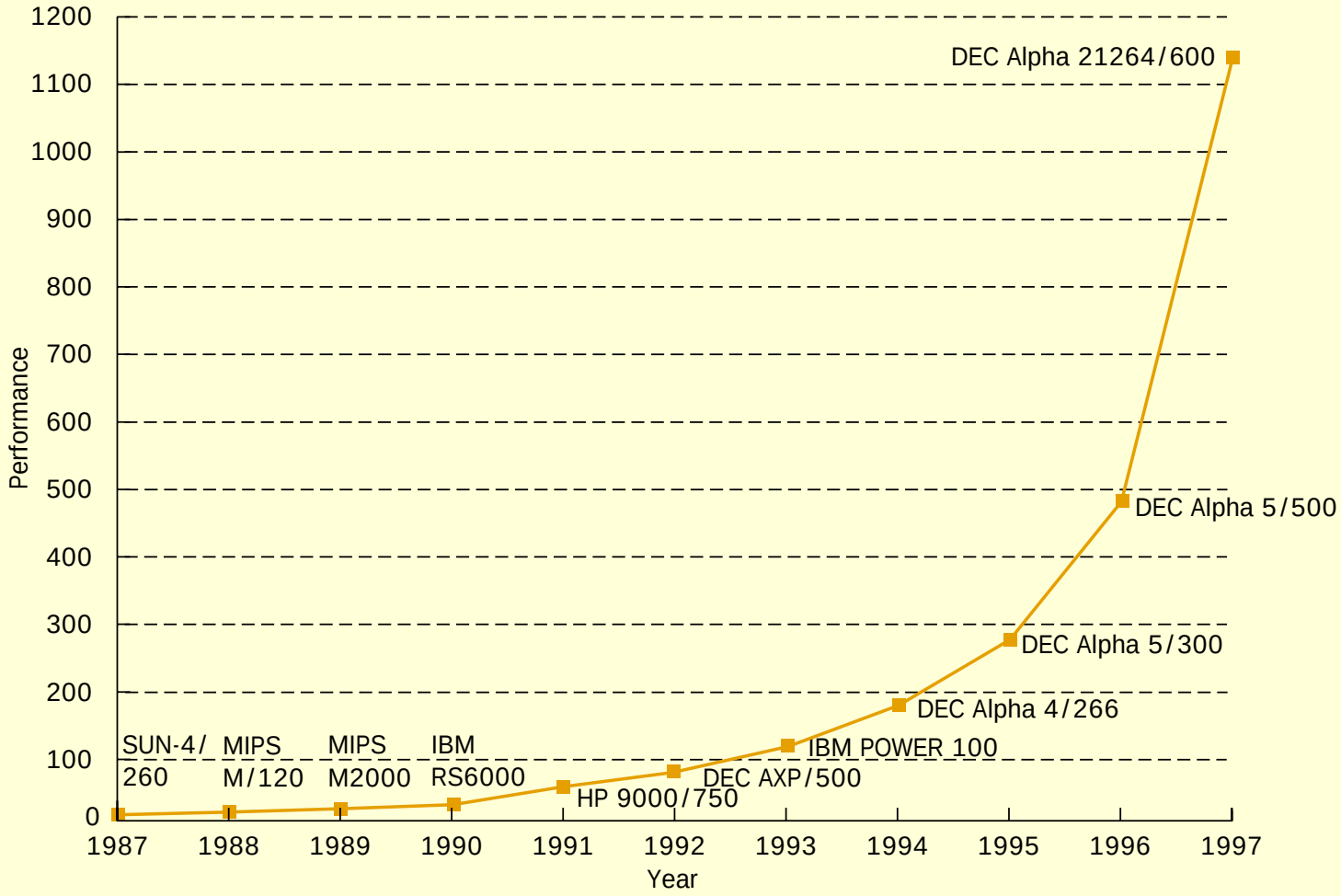
- Changing something by a factor of 30 usually makes a *qualitative* change.
  - 10 people/square mile – farmland
  - 300 p/mi^2 – suburbia
  - 10,000 p/mi^2 – big city

  - 2 mph – walking
  - 60 mph – driving
  - 900 mph – fast jet

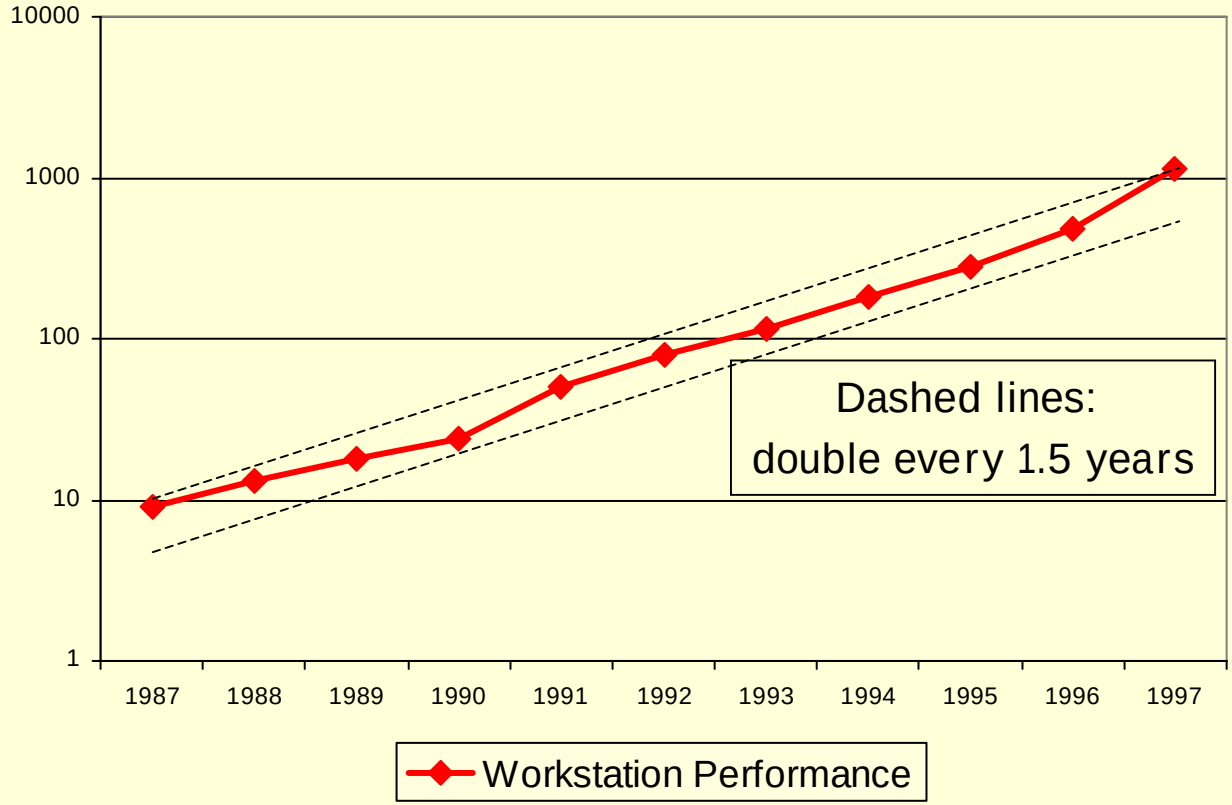
  - 100 square feet – dorm room
  - 3000 ft^2 – good sized house
  - 100,000 ft^2 – hotel

- Computers are making such changes every decade (or faster).

Performance Trends



Performance on a logarithmic scale ...



Memory Evolution

- Transistors get smaller, chips get larger, results is ...
- DRAM chip capacity doubles every 1.5 years

Processor Evolution

- Transistor count doubles every 2 years
- Clock speed doubles every 2 years
- Memory speeds increase a tiny bit

  and then a miracle occurs ...

- Performance doubles every 1.5 years

Why is this surprising?

- You'd expect processor speed to increase with the slowest technology
  - P&H estimate DRAM access time decreases 9% per year.
  - So memory speed doubles about every ??? years.
- Yet performance doubles *faster* than clock speed!
  - And *much* faster than DRAM access time.
- Somehow, added transistors are being used effectively.
  - Sure, maybe you can wash your car faster with 4 people.
  - But does having 16 or 64 people help???
- Later part of course is about how this happens!

When will computer exceed brain?

- Human brain has about 10 billion neurons.
- Each is connected to about 100,000 other neurons.
- A neuron can "fire" about 1000 times/sec.
- Estimate when microprocessors will exceed a human's brainpower.
  - Assume each "fire" decision corresponds to computation of 100,000 transistors (one per connection) for one clock cycle.

Machine of the day – Babbage's engines

- Difference engine
  - Started 1847; completed 1991 (London Science Museum)
  - Very big calculator – 11 feet long, 7 feet high, 4000 parts
- Analytical engine
  - Had "Mill" (processor) and "Store" (memory)
  - Had conditional branches; could execute loops
  - Op code and address written on separate punch cards
- Neither machine completed by Babbage
  - Small analytical engine built by his son; calculated multiples of pi (incorrectly)
- First programmer: Ada Lovelace