Illumination: The Why, the What and the How

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Why is it hard?

1. Each Scene is a dynamical system.
2. Objects interact with their environments.
3. The presence of a new object changes the energy balance in the scene.
4. The problem is under constrained.

Direct Illumination

Global Illumination

The Solution

- The Distant Scene
- The Local Scene
- The Synthetic Objects

Distant Scene

- A Model of incoming light.
- Contributes direct illumination only.
- The reflections in the scene do not affect the distant illumination.
- Modelled using a envionment map.

Local Scene

- A model of the local geometry of the scene.
- A model of the local reflectance in the scene estimated from the scene or prior knowledge.
- Contributes direct as well as indirect illumination to the synthetic objects.

Synthetic Objects

- A model of the geometry of the objects
- A model of the reflectance of the object

The Method

- Capture the distant illumination in the scene.
- Measure the local BRDF.
- Model the synthetic objects and the local geometry.
- Render the objects and the local scene using full global illuminations.
- Composit into the original scene using differential rendering.

Anatomy of a light source

- What is a light source?
- What is the space of all light sources?
- How can we move about in this space ?

Definitions

Radiance

Radiance is the amount of energy per unit time per unit solid angle per unit area in the direction of travel.

or

The number of photons striking a point from a particular direction per second.

Radiance remains constant along a line in free space.

Definitions

Free Space $(\mathcal{F})$
A bounded, open connected subset of 3d eucledian space. $\partial\mathcal{F}$ is the boundary of $\mathcal{F}$.

Set of rays $(\mathcal{M}(\mathcal{F}))$
The set of all closed directed lines $[\mathbf{x_1}, \mathbf{x_2}]$ s.t.

1. $\mathbf{x_1} \neq \mathbf{x_2}$
2. $\mathbf{x_1}, \mathbf{x_2} \in \partial\mathcal{F}$
3. The line joining $\mathbf{x_1}$ and $\mathbf{x_2}$ is contained entirely in $\mathcal{F}$.

Ray Manifold

Given a z_0, let $\mathbf{r}$ be a ray which passes through $\mathbf{x_0} = \{x_0, y_0, z_0\}$, in the direction $(p_0, q_0, 1)$, then we make the association

$$\mathbf{r} \longrightarrow (x_0, y_0, p_0, q_0)$$

Ray Manifold : Given a free space $\mathcal{F}$, the set of rays $\mathcal{M}(\mathcal{F})$ is a 4-D manifold

Ray Manifold

$$R : \mathcal{M}(\mathcal{F}) \rightarrow [0, \infty)$$

R is the radiance along a ray $\mathbf{r}$. Radiance remains constant along in a line in free space.

$$R_{z_0}(x_0, y_0, p_0, q_0) = R_{z_1}(x + (z_1 - z_0)p_0, y_0 + (z_1 - z_0)q_0, p_0, q_0)$$

The Lightsource Hypercube

Given a plane $\mathcal{P}_{z_0}$, consider the set of rays

$$\begin{aligned}\mathcal{M} = (x, y, p, q) \quad : \quad & x \in [\frac{h_x}{2}, \frac{h_x}{2}], \\ & y \in [-\frac{h_y}{2}, \frac{h_y}{2}] \\ & p \in [-\frac{h_p}{2}, \frac{h_p}{2}], \\ & q \in [-\frac{h_q}{2}, \frac{h_q}{2}]\}\end{aligned}$$

each having uniform radiance, $R(h_x, h_y, h_p, h_q)$.

Lightsource Hypercube

Some integration shows that the radiant flux from this set of rays is

$$\Phi = h_x h_y R(h_x, h_y, h_p, h_q) \int_{-h_p/2}^{h_p/2} \int_{-h_q/2}^{h_q/2} \frac{dp dq}{(1+p^2+q^2)^2}$$

set

$$R(h_x, h_y, h_p, h_q) = \frac{1}{h_x h_y} \left[\int_{-h_p/2}^{h_p/2} \int_{-h_q/2}^{h_q/2} \frac{dp dq}{(1+p^2+q^2)^2} \right]^{-1}$$

so that

$$\Phi = 1$$

Lightsource Hypercube

Let $I(\cdot)$ denote the indicator function on the interval $[-1/2, 1/2]$. A uniform cubic source of unit flux centered at position $(0, 0, z_0)$ is a source with the radiance function

$$R_{z_0}(x, y, p, q) =$$
$$R(h_x, h_y, h_p, h_q) I\left(\frac{x}{h_x}\right) I\left(\frac{y}{h_y}\right) I\left(\frac{p}{h_p}\right) I\left(\frac{q}{h_q}\right)$$

The set of all light sources can now be obtained by restricting the coordinates in various manners.

Light Sources

Real Source	Ideal model	h_x	h_y	h_p	h_q
Overcast Sky	Uniform Source	∞	∞	∞	∞
Laser	Single Ray	0	0	0	0
Flourescent Tube	Linear Source	∞	0	∞	∞
Sunlight	Directed Point Source	∞	∞	0	0

Light Sources

Real Source	Ideal model	h_x	h_y	h_p	h_q
Louveres	Fan of rays	∞	0	0	∞
Small Panel Light	Point Source	0	0	∞	∞
Light through crack	Parallel Rays	∞	0	0	0
Rotating spotlight	Fan of rays	0	0	∞	0

Source Rays

That minimal subset $\mathcal{M}_{src} \subseteq \mathcal{M}(\mathcal{F})$, s.t. if they are removed, the radiance on the manifold $\mathcal{M}(\mathcal{F})$ would be identically zero.

Radiance Map

St. Peter's Basilica

An omni-directional, high dynamic range image that records the incident illumination conditions at a particular point in space.

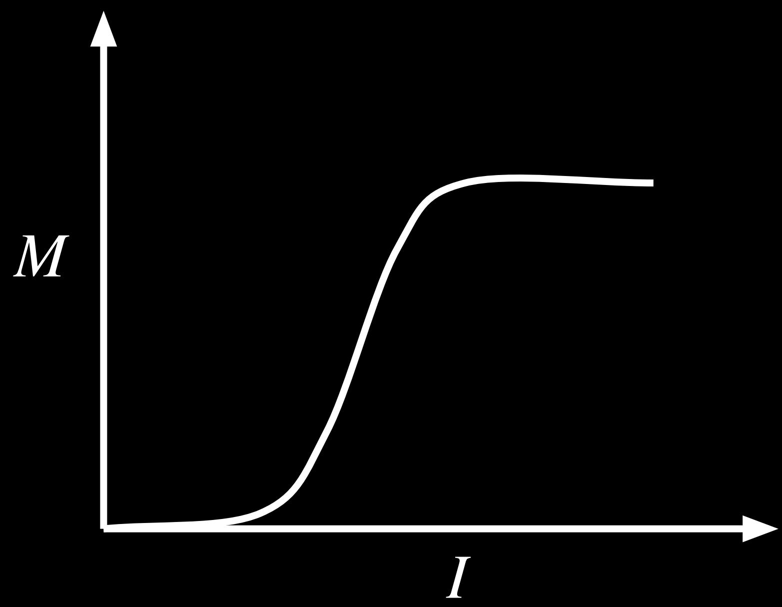
Real Pixels are floats

$$
\begin{aligned}
I &= L\frac{\pi}{4}\left(\frac{d}{h}\right)^2 \cos^4\phi t && (1)\\
&= LPe && (2)
\end{aligned}
$$

L : Scene Radiance
$e = \frac{\pi d^2}{4} t$: exposure.

There is no bound on the magnitude of I.

The radiometric response function



$$M = f(I)$$

- M is the observed image brightness.
- M is bounded with finite dynamic range.
- Estimating I requires estimating $g = f^{-1}$.

Estimating g

- Use multiple exposures to estimate g.
- Non-Parametric Regression - Debevec & Malik
- Parametric Regression - Mitsunaga & Nayar

Debevec & Malik

- The range of f is discrete and finite.
- f is monotonic and smooth.

$$\begin{aligned} M_{i,j} &= f(I_{i,j}) && (3) \\ g(M_{i,j}) &= I_{i,j} && (4) \\ g(M_{i,j}) &= L_i P_i e_j && (5) \\ \log g(M_{i,j}) &= \log L_i + \log P_i + \log e_j && (6) \end{aligned}$$

Non-Parametric Regression

$$\mathcal{O} = \sum_i \sum_j \left[g(M_{i,j}) - \log L_i P_i - \log e_j\right]^2 + \lambda \sum_z g''(z)^2$$

$$g''(z) = g(z-1) - 2g(z) + g(z+1)$$

Mitsunaga & Nayar

- Assume a flexible polynomial model.
- Perform regression to estimate the parameters of the mode.

$$I_{i,j} = g(M_{i,j}) = \sum_{n=0}^{N} c_n M_{i,j}^n$$

Mitsunaga & Nayar

$$\frac{I_{i,j}}{I_{i,j+1}} = \frac{L_i P_i e_j}{L_i P_i e_{j+1}} = R_{q,q+1} = \frac{g(M_{i,j})}{g(M_{i,j+1})}$$

$$\mathcal{O} = \sum_i \sum_j \left[\sum_n c_n M_{i,j}^n - R_{j,j+1} \sum_n c_n M_{i,j+1}^n \right]^2$$

and

$$\sum_n c_n = I_{max}$$

Re-estimating Exposure Ratios

$$R_{j,j+1}^{(k)} = \frac{1}{N} \sum_{i=1}^{N} \frac{\sum_n c_n^{(k)} M_{i,j}^n}{\sum_n c_n^{(k)} M_{i,j+1}^n}$$

Calculating the HDR Image

$$\log I_i = \frac{\sum_j w(M_{i,j})(g(M_{i,j}) - \log e_j)}{\sum_j w(M_{i,j})}$$

$$w(z) = \begin{cases} z - Z_{min} \text{ for } z \leq \frac{1}{2}(Z_{min} + Z_{max}) \\ Z_{max} - z \text{ for } z > \frac{1}{2}(Z_{min} + Z_{max}) \end{cases}$$

$$w(z) = \frac{g(z)}{g'(z)}$$

Estimating g: a second look

$$M_A = f(I_A) = f(LPe_A)$$

$$M_B = f(I_B) = f(LPe_B)$$

$$\frac{I_A}{e_A} = \frac{I_B}{e_B} = LP$$

or

$$g(M_B) = kg(M_A), \quad k = e_B/e_A$$

Brightness Transfer Function

$$M_B = T(M_A) = g^{-1}(kg(M_A))$$

The brightness transfer function relates the brightness change from one image to the other as the exposure changes.

Properties of T

If g is smooth and monotonically increasing with a smooth inverse. $g(0) = 0, g(1) = 1$ and $k > 1$, then

- T is monotonically increasing.
- $T(0) = 0$
- $T(M) \geq M$
- $\lim_{n\to\infty} T^{-n}(M) = 0$

Fractal Ambiguity

$$g(T(M)) = kg(M)$$

if $T(M) \in [a, b]$, then the above equation relates

$$g([a, b]) \leftrightarrow g([T^{-1}(a), T^{-1}(b)])$$

in the case of $[a, b] = [T^{-(1}(1), 1]$, it is

$$g([T^{-1}(1), 1]) \leftrightarrow g([T^{-2}(1), T^{-1}(1)])$$

$$g([T^{-n}(1), T^{-(n-1)}(1)]) \leftrightarrow g([T^{-(n+1)}(1), T^{-n}(1)])$$

Fractal Ambiguity

- The above equations constrain the behaviour of g on $[0, T^{-1}(1))$.
- The behaviour on $[T^{-1}(1), 1]$ is underconstrained.
- We can choose an abitrary smooth monotonic function s, s.t $s(1) = 1$, and $s(T^{-1}(1)) = 1/k$ and extend it to a solution g.
- Debevec & Mailk solved the ambiguity by imposing a smoothness constraint.
- Mitsunaga & Nayar constrained the solution to be a polynomial.

Exponential Ambiguity

What if, k as well as g are both unknown ?

$$
\begin{aligned}
g(T(M)) &= kg(M) \\
[g(T(M))]^{\gamma} &= [kg(M)]^{\gamma} \\
g^{\gamma}(T(M)) &= k^{\gamma}g^{\gamma}(M)
\end{aligned}
$$

Hence is (g, k) is a solution then (g^{γ}, k^{γ}), $\gamma > 0$ is also a solution.

g and k cannot be jointly estimated.

Recovering the Exposure Ratio

It is possible to recover the exposure ratio of two images without knowing the response function g.

$$g(T(M)) = kg(M) \tag{7}$$
$$g'(T(M))T'(M) = kg'(M) \tag{8}$$

If g’(0) ¿ 0 , then

$$k = T'(0)$$

What happened to the exponential ambiguity?

Estimation without registration

- Previous approaches were based on exact pixel correspondences.
- Difficult getting data which is perfectly aligned.
- Is it possible to recover the transfer function without exact pixel correspondence?

Estimation without registration

H_A and H_B are empirical distribution functions of image brightness values.
$H_A(u)$ = # of points in A with brightness less than or equal to u.
Hence,
$H_B(T(u))$ = # of points in B with brightness less than or equal to $T(u)$
= # of points in A with brightneess less than or equal to u.

$$H_B(T(u)) = H_A(u)$$

$$T(u) = H_B^{-1}(H_A(u))$$

Back to inserting objects

Estimating local scene BRDF

1. Assume reflectance model.
2. Render the local scene.
3. Compare computed with actual.
4. Adjust paramerts of reflectance model.
5. Goto 2.

Differential Rendering

1. Render the local scene L_{local}
2. Calculate difference between the actual and the computed.

$$L_{error} = L_{actual} - L_{local}$$

3. Render the local scene with the synthetic objects L_{object}
4. Composit using the error.

$$L_{final} = L_{actual} + (L_{object} - L_{error})$$

RNL

Fiat Lux

References

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