Background: Camera models, Transforms, Radiometry

Topics in Image-Based Modeling and Rendering

CSE291 J00

Lecture 2

Outline

- Camera Models
- Pinhole perspective
- Affine/Orthographic models
- Homogeneous coordinates
- Coordinate transforms
- Lenses
- Radiometry
  - Irradiance
  - Radiance
  - BRDF

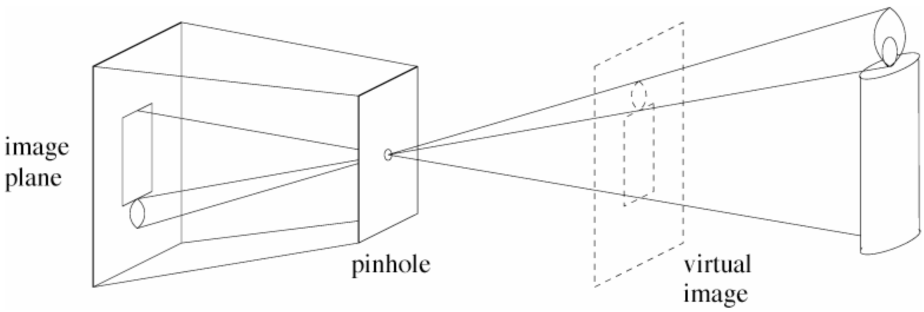
Effect of Lighting: Monet

Change of Viewpoint: Monet

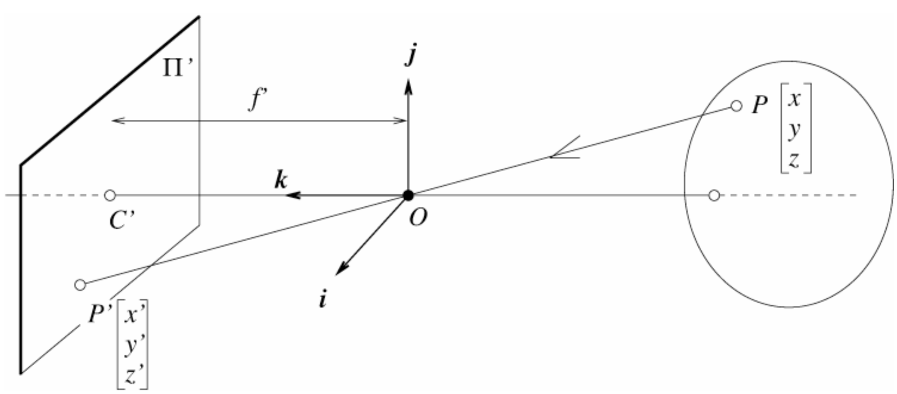
Haystack at Chailly at sunrise(1865)

Pinhole cameras

- Abstract camera model - box with a small hole in it
- Pinhole cameras work in practice



The equation of projection



Cartesian coordinates:

- We have, by similar triangles, that (x, y, z) -> (f x/z, f y/z, -f)
- Ignore the third coordinate, and get

$$(x,y,z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Homogenous coordinates

- Add an extra coordinate and use an equivalence relation
- for 2D
  - equivalence relation k*(X,Y,Z) is the same as (X,Y,Z)
- for 3D
  - equivalence relation k*(X,Y,Z,T) is the same as (X,Y,Z,T)
- Basic notion
  - Possible to represent points “at infinity”
    - Where parallel lines intersect
    - Where parallel planes intersect
  - Possible to write the action of a perspective camera as a matrix

Euclidean -> Homogenous-> Euclidean

In 2-D

- Euclidean -> Homogenous: (x, y) -> k (x,y,1)
- Homogenous -> Euclidean: (x, y, z) -> (x/z, y/z)

In 3-D

- Euclidean -> Homogenous: (x, y, z) -> k (x,y,z,1)
- Homogenous -> Euclidean: (x, y, z, w) -> (x/w, y/w, z/w)

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The camera matrix

Turn previous expression into Homogenous Coordinates
- HC's for 3D point are (X,Y,Z,T)
- HC's for point in image are (U,V,W)

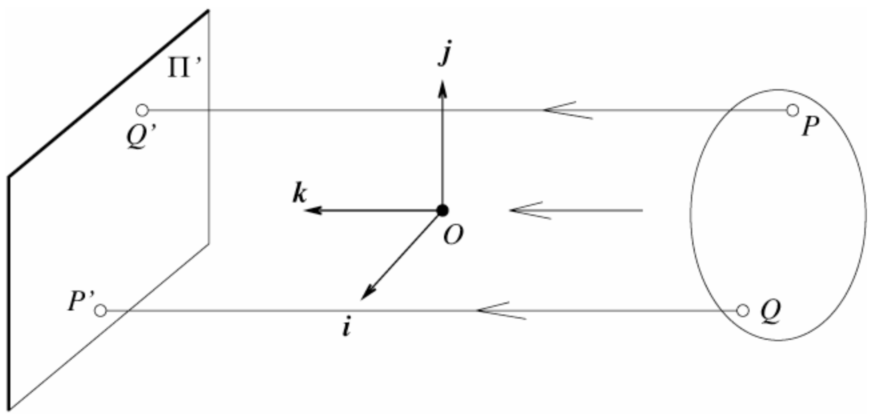
$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about (some point (x_0, y_0, z_0).
- Drop terms of higher order than linear.
- Resulting expression is affine camera model

Orthographic projection

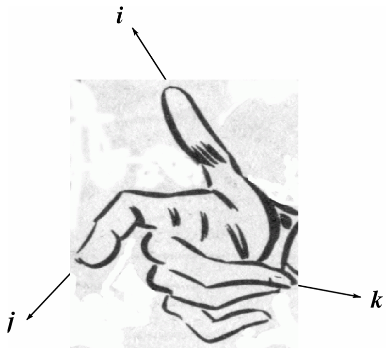
Take Taylor series about $(0, 0, z_0)$ – a point on optical axis

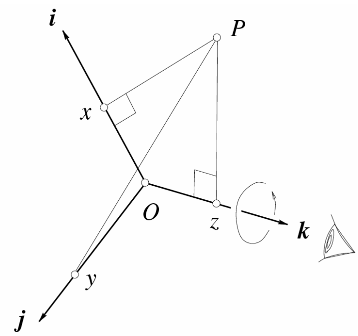


The projection matrix for orthographic projection

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

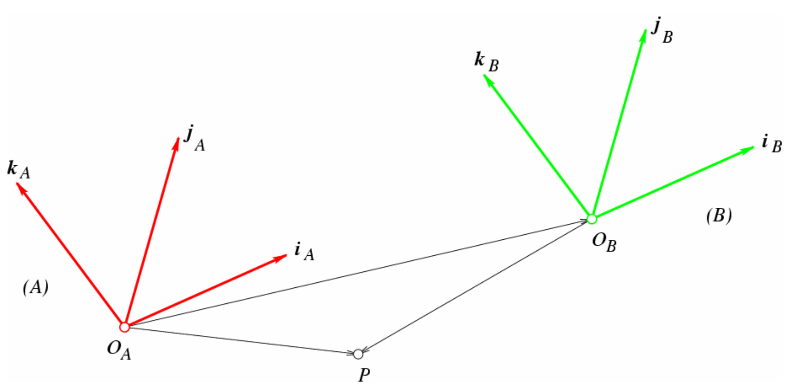
Euclidean Coordinate Systems





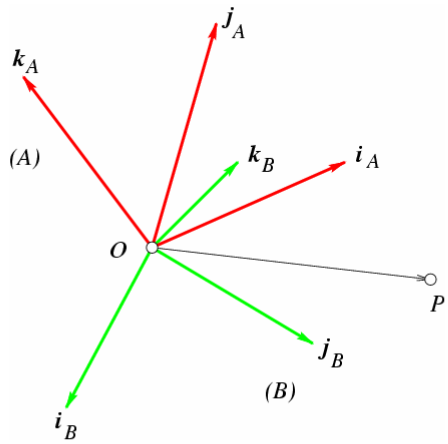
$$\begin{cases} x = \overrightarrow{OP}.\mathbf{i} \\ y = \overrightarrow{OP}.\mathbf{j} \\ z = \overrightarrow{OP}.\mathbf{k} \end{cases} \Leftrightarrow \quad \overrightarrow{OP} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \quad \Leftrightarrow \quad \mathbf{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Coordinate Changes: Pure Translations



$$O_BP = O_BO_A + O_AP \quad , \quad {}^BP = {}^AP + {}^BO_A$$

Rotation Matrix



$$
{}^B_A R = \begin{bmatrix} \mathbf{i}_A.\mathbf{i}_B & \mathbf{j}_A.\mathbf{i}_B & \mathbf{k}_A.\mathbf{i}_B \\ \mathbf{i}_A.\mathbf{j}_B & \mathbf{j}_A.\mathbf{j}_B & \mathbf{k}_A.\mathbf{j}_B \\ \mathbf{i}_A.\mathbf{k}_B & \mathbf{j}_A.\mathbf{k}_B & \mathbf{k}_A.\mathbf{k}_B \end{bmatrix} = \begin{bmatrix} {}^A\mathbf{i}_B^T \\ {}^A\mathbf{j}_B^T \\ {}^A\mathbf{k}_B^T \end{bmatrix} = \begin{bmatrix} {}^B\mathbf{i}_A & {}^B\mathbf{j}_A & {}^B\mathbf{k}_A \end{bmatrix}
$$

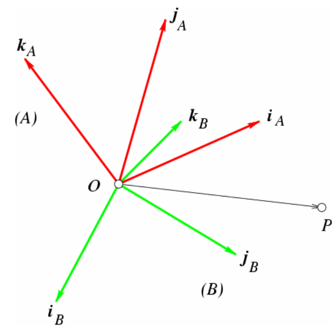
A rotation matrix is characterized by the following properties:

- Its inverse is equal to its transpose, and
- its determinant is equal to 1.

Or equivalently:

- Its rows (or columns) form a right-handed orthonormal coordinate system.

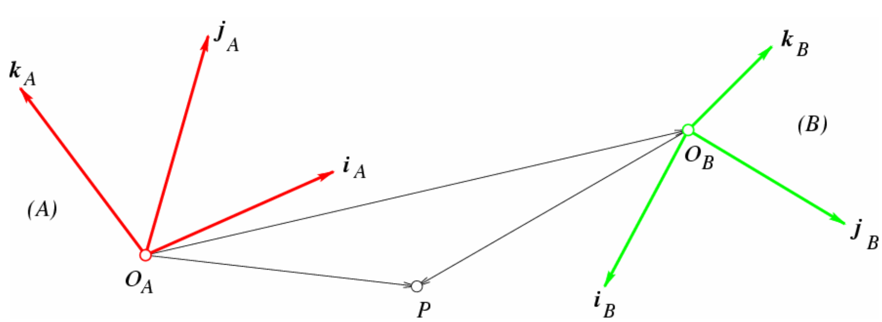
Coordinate Changes: Pure Rotations



$$\vec{OP} = [\mathbf{i}_A \quad \mathbf{j}_A \quad \mathbf{k}_A] \begin{bmatrix} {}^A x \\ {}^A y \\ {}^A z \end{bmatrix} = [\mathbf{i}_B \quad \mathbf{j}_B \quad \mathbf{k}_B] \begin{bmatrix} {}^B x \\ {}^B y \\ {}^B z \end{bmatrix}$$

$$\Rightarrow \quad {}^B P = {}^B_A R \, {}^A P$$

Coordinate Changes: Rigid Transformations



$${}^B P = {}^B_A R \, {}^A P + {}^B O_A$$

Block Matrix Multiplication

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

What is AB ?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Homogeneous Representation of Rigid Transformations

$$\begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{B}_{A}R & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{B}_{A}R\,{}^{A}P + {}^{B}O_{A} \\ 1 \end{bmatrix} = {}^{B}_{A}T \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}$$

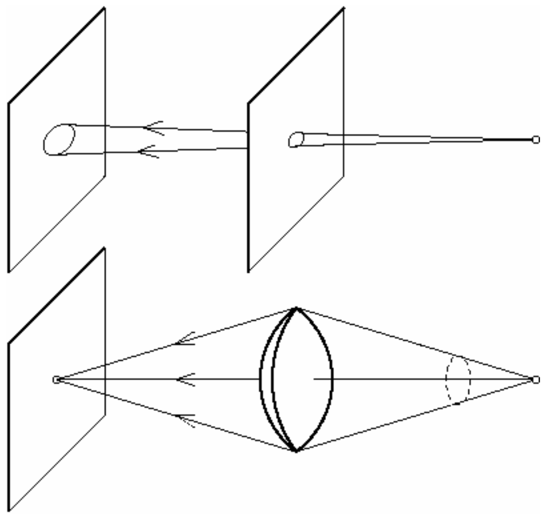
Camera parameters

- Issue
  - camera may not be at the origin, looking down the z-axis
    - extrinsic parameters
  - one unit in camera coordinates may not be the same as one unit in world coordinates
    - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{intrinsic parameters} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{extrinsic parameters} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

3 x 3 4 x 4

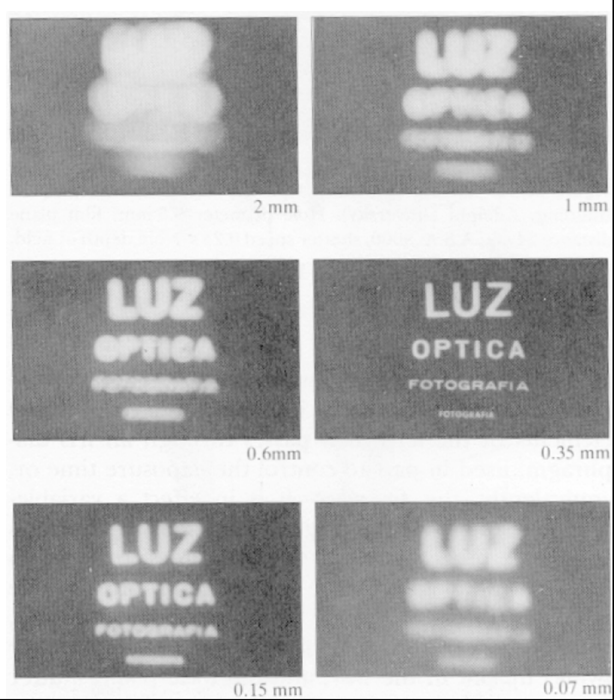
The reason for lenses



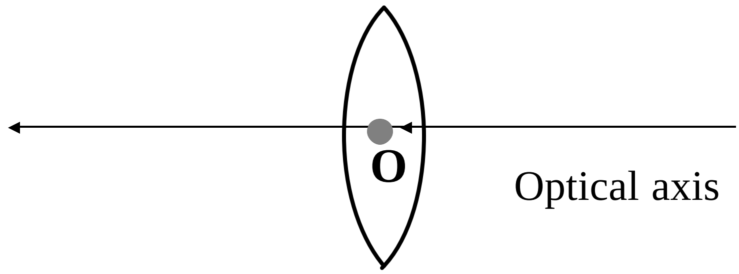
Pinhole too big -
many directions are averaged, blurring the image

Pinhole too small-
diffraction effects blur the image

Generally, pinhole cameras are *dark*, because a very small set of rays from a particular point hits the screen.

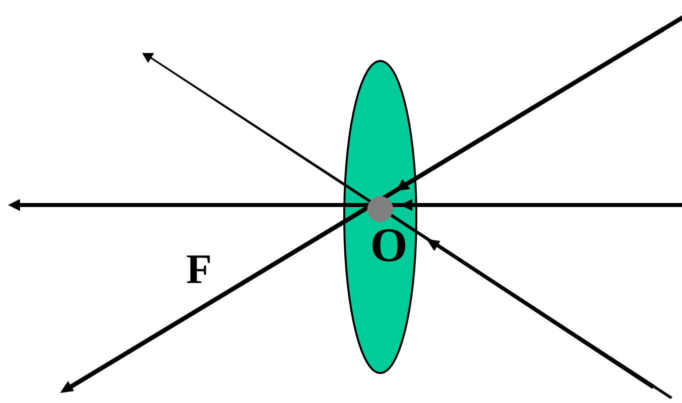


Thin Lens



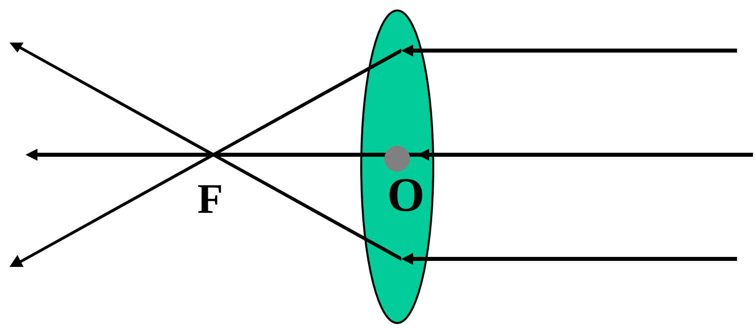
- Rotationally symmetric about optical axis.
- Spherical interfaces.

Thin Lens: Center



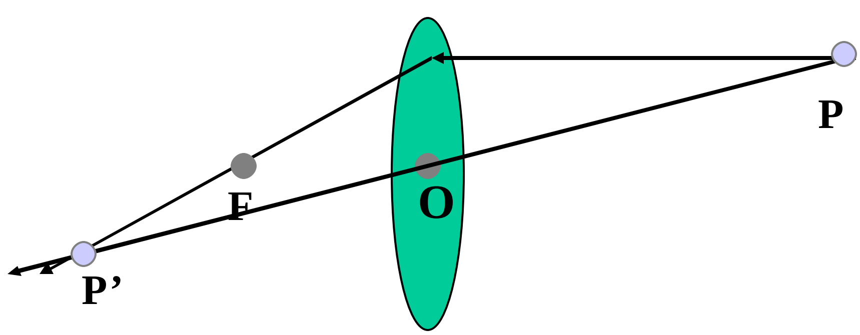
- All rays that enter lens along line pointing at **O** emerge in same direction.

Thin Lens: Focus



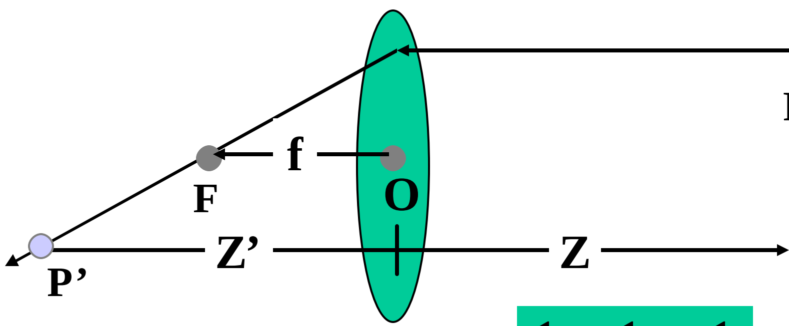
Parallel lines pass through the focus, F

Thin Lens: Image of Point



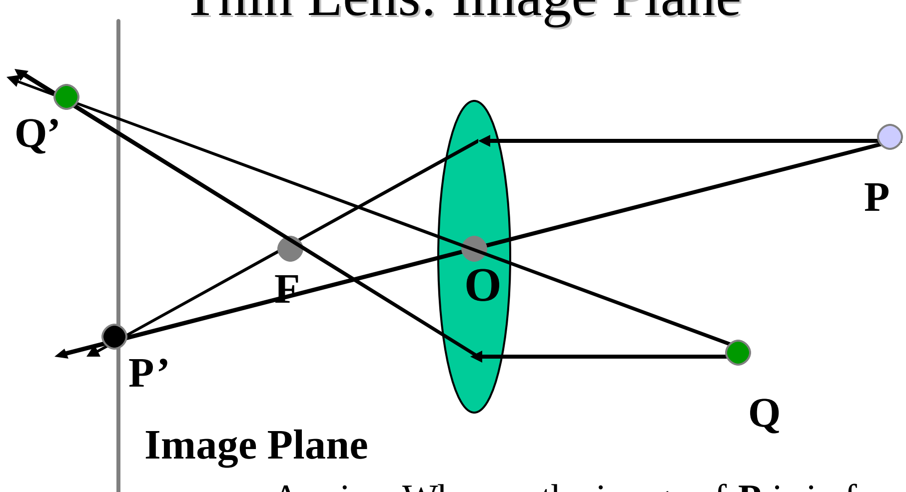
All rays passing through lens and starting at **P** converge upon **P'**

Thin Lens: Image of Point



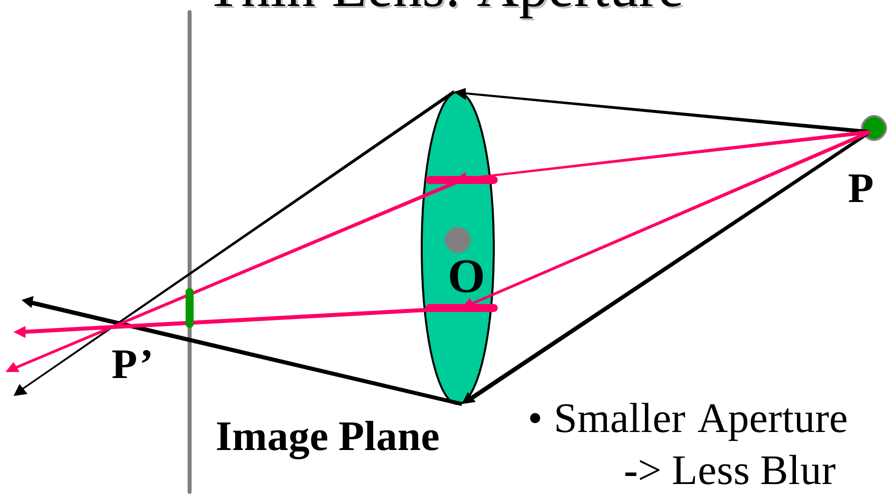
$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

Thin Lens: Image Plane



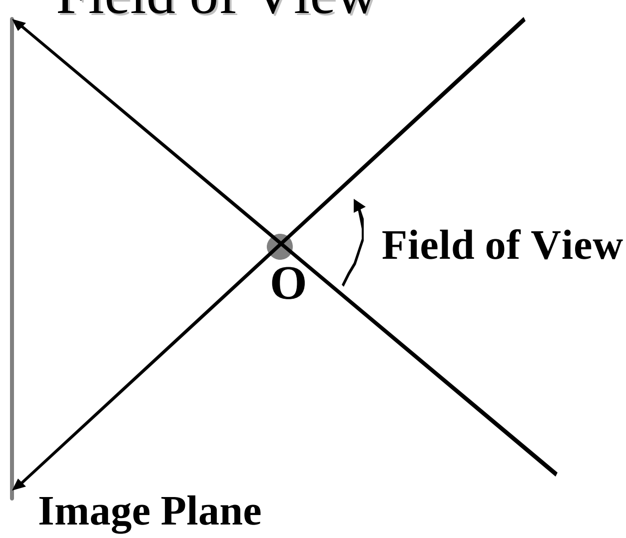
A price: Whereas the image of **P** is in focus, the image of **Q** isn't.

Thin Lens: Aperture



- Smaller Aperture -> Less Blur
- Pinhole -> No Blur

Field of View



Earliest Surviving Photograph

- First photograph on record, “la table service” by Nicephore Niepce in 1822.
- Note: First photograph by Niepce was in 1816.

Radiometry

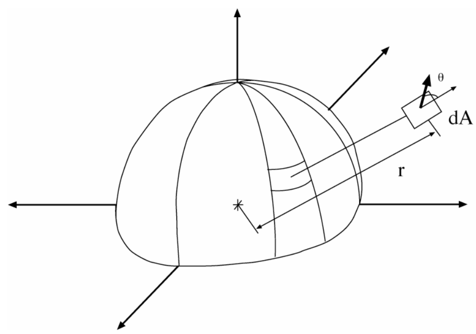
- Solid Angle
- Irradiance
- Radiance
- BRDF
- Lambertian/Phong BRDF

Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point

- The solid angle subtended by a patch area dA is given by

$$dw = \frac{dA\cos J}{r^2}$$



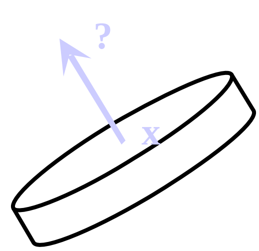
Radiance

- Power is energy per unit time

- **Radiance: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle**

- Symbol: L(x,?)

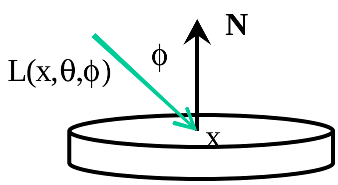
- Units: watts per square meter per steradian ($wm^{-2}sr^{-1}$)



Irradiance

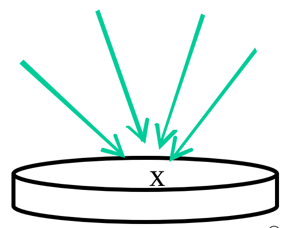
- How much light is arriving at a surface?
- Sensible unit is *Irradiance*
- Incident power per unit area *not foreshortened*
- This is a function of incoming angle.
- A surface experiencing radiance L(x,θ,φ) coming in from solid angle dω experiences irradiance:

$$L(\underline{x},q,f)\cos f\,dw$$



- Crucial property:
  Total power arriving at the surface is given by adding irradiance over all incoming angles Total power is

$$\int_{\Omega} L(\underline{x},q,f)\cos f\sin q\,dq\,df$$



Camera's sensor

- Measured pixel intensity is a function of irradiance integrated over
  - pixel's area
  - over a range of wavelengths
  - For some time
- Ideally, it's proportional to the radiance.

$$I = \int_t \int_l \int_x \int_y E(x,y,l,t)s(x,y)q(l)\,dy\,dx\,dl\,dt$$

Light at surfaces

Many effects when light strikes a surface -- could be:

- transmitted
  - Skin, glass
- reflected
  - mirror
- scattered
  - milk
- travel along the surface and leave at some other point
- absorbed
  - sweaty skin

Assume that

- surfaces don’t fluoresce
  - e.g. scorpions, detergents
- surfaces don’t emit light (i.e. are cool)
- all the light leaving a point is due to that arriving at that point

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BRDF

With assumptions in previous slide

- Bi-directional Reflectance Distribution Function

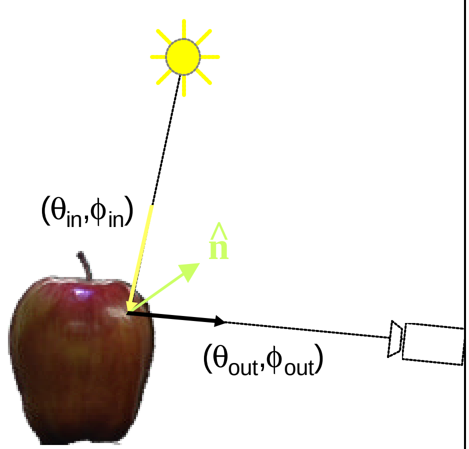
  $\rho(\theta_{in}, \phi_{in} ; \theta_{out}, \phi_{out})$

- **Ratio of incident irradiance to emitted radiance**

- Function of
  - Incoming light direction:

    θ_{in} , ϕ_{in}
  - Outgoing light direction:

    $\theta_{out} , \phi_{out}$



$$r\ (\underline{x};q_{in},f_{in};q_{out},f_{out}) = \frac{L_o(\underline{x};q_{out},f_{out})}{L_i(\underline{x};q_{in},f_{in})\cos f_{in} dw}$$

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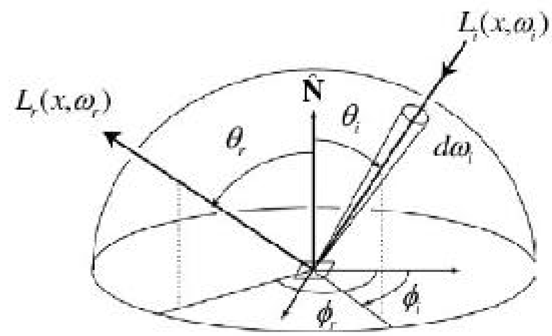
The Reflection Equation



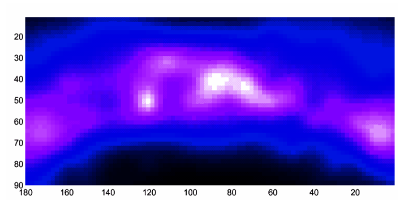
$$L_r(x,\omega_r) = \int_{H^2} f_r(x,\omega_i \rightarrow \omega_r) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

Surface Reflectance Models

Common Models

- Lambertian
- Phong
- Physics-based
  - Specular
    [Blinn 1977], [Cook-Torrance 1982], [Ward 1992]
  - Diffuse
    [Hanrahan, Kreuger 1993]
  - Generalized Lambertian
    [Oren, Nayar 1995]
  - Thoroughly Pitted Surfaces
    [Koenderink et al 1999]
- Phenomenological
  [Koenderink, Van Doorn 1996]

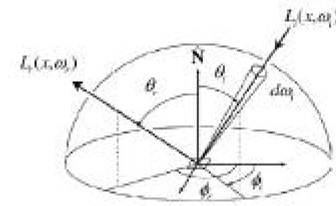
Arbitrary Reflectance



- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement *[Dana et al, 1999], [Marschner]*

Lambertian (Diffuse) Surface

- BRDF is a constant called the albedo
- Emitted radiance is NOT a function of outgoing direction – i.e. constant in all directions.
- For lighting coming in single direction, emitted radiance is proportional to cosine of the angle between normal and light direction

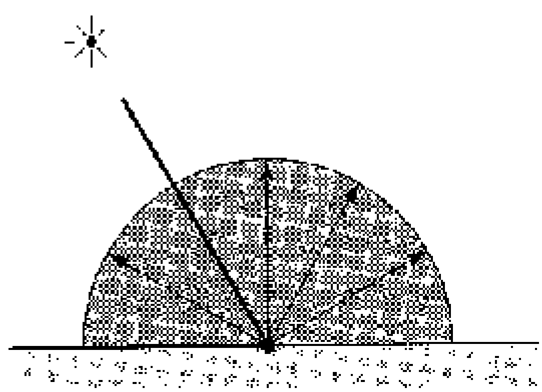


$$L_r(x,\omega_r) = \int_{H^2} f_r(x,\omega_i \to \omega_r) L_i(x,\omega_i)\cos\theta_i \, d\omega$$

$$L_r = N . \omega_\iota$$

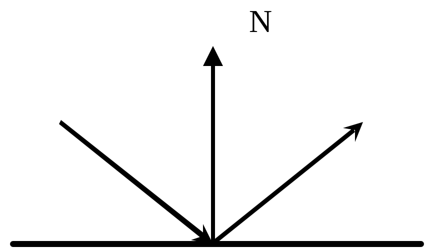
Lambertian reflection

- Lambertian
- Matte
- Diffuse



Light is radiated equally in all directions

Specular Reflection: Smooth Surface



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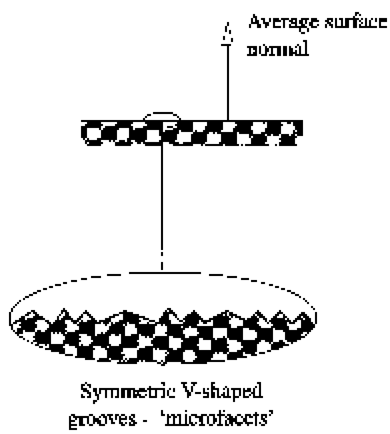
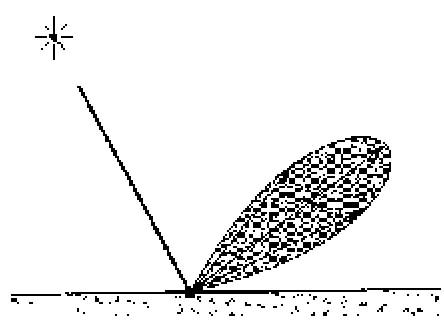
Rough Specular Surface





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Phong Model